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Assessing Income Distribution Using Kernel Estimates: A Comparative Study in Five European Countries

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Document n° 27 **Assessing Income Distribution Using Kernel Estimates:
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By Christos Papatheodorou, Paraskevi Peristera, Anastasia Kostaki

ASSESSING INCOME DISTRIBUTION USING KERNEL ESTIMATES: A COMPARATIVE STUDY IN FIVE EUROPEAN COUNTRIES

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Abstract

This paper compares and assesses the income inequality between five European countries in the mid 1990's, employing the non-parametric technique of kernel density estimation. The countries used in this inequality exercise were Germany, Hungary, Luxembourg, Poland and the UK, and the analysis was based on comparative data and variables provided by the PACO project. Kernel density estimates were found particular revealing for comparing the shape of income distributions between populations, and for mapping the impact that differences in income polarisation and concentration in various subgroups have on the overall income distribution of a country.

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1 Introduction

The main aim of this paper is to compare and assess the income inequality between five European countries employing the non-parametric technique of kernel density estimation. The countries used in this inequality exercise are Germany, Hungary, Luxembourg, Poland and the UK, which are representing developed economies and economies in transition with different social structures, social security systems and degree of inequality. The analysis is based on comparative data and variables provided by the PACO (Panel Comparability) project.

In the last decades the comparisons of income inequality between countries has gained an increasing significance for social research and for the relevant policy debate. Cross national comparisons are considered to provide valuable information for effective policy evaluation and interventions, since they allow the investigation of the impact that certain social structures and main institutions, such as the labour market, the social security system and the welfare regime, have on the way that incomes are distributed between persons. However, such inequality exercises are often subject to certain restrictions imposed by the particular indices or summary measures used, which could puzzle or misguide those who are not familiar with their properties. None of the alternative indices that have been proposed for measuring and assessing inequality can be considered as value free (Atkinson 1983, Cowell 2000, 1995, Sen 1997). Each of these summary measures weights the transfers differently at various points of the income scale and thus imposes (explicitly or implicitly) certain value judgments about society. Thus even an expert in the field could experience difficulties in drawing conclusions about the exact shape and the main characteristics of an income distribution based solely on findings from certain inequality indices. Graphical representations, such as histograms, Lorenz curves and Pen's Parades have been suggested and frequently used as alternative or complementary tools for summarising income distributions. These graphs do not simply focus on one specific feature of the distribution, as the various summary measures do, but they summarise various characteristics of the whole distribution. Within this framework we argue that kernel density functions and the corresponding graphs, provide a simple and straightforward interpretation of the income distribution. Kernel density estimates, recently employed for investigating inter-temporal changes of income distribution within countries, could also be quite revealing for comparative analysis between countries. They could help capture differences between populations in the concentration at various parts of the income scale and investigate issues related to income polarisation between certain population groups.

Generally, the shape of the distribution of income for a population can be viewed as a weighted averaging of the underlined distributions in certain population subgroups. Indeed, empirical evidence has shown that there are significant income differences between certain population subgroups, and a number of theories have emphasised certain attributes in explaining the income disparities between people. Two main questions, both crucial for policy makers, emerge from these positions. First, to what extent could differences in income disparities between certain population subgroups explain the shape of the distribution of income of the total population. Second, to what extent are there similarities and differences between countries in the way that income is distributed between certain population subgroups. We investigate these questions by providing kernel density estimates for two population subgroups formed according to age (elderly and non elderly) and educational level. Both characteristics are generally acknowledged as important factors in determining people's income. Kernel density estimates are considered particularly helpful in mapping the impact that differences in income polarisation and concentration in various subgroups have on overall income distribution of a country.

The structure of the rest of the paper is set out as follows. Section 2 is devoted to a presentation of the kernel density estimation. The data used for the analysis and the adopted variable and data definitions are discussed in Section 3. In Section 4 the main findings are demonstrated and discussed. Estimates on inequality based on certain summary measures are also presented and issues related to the restriction imposed by these estimates in comparing inequality between populations are discussed. Finally, in Section 5 we summarise the main findings and provide some concluding remarks. The definitions and formulae of the inequality indices and the summary measures used are presented in the Appendix.

2 Kernel density estimation

An extensively used, extremely simple and effective tool to explore the shape of the frequency distribution of any given data set is the histogram. Such a presentation simply provides a rough indication of the form of the underlying density function. For doing that the range of the data set considered is divided in subintervals and then every single observation is substituted by the midpoint value of the interval in which the observation lies. The areas of each box in the histogram are analogous to the corresponding observed frequencies. However, such a representation, though in a lower degree than the original values, bears the effect of random variations incorporated in the observations preventing the user to perceive the underlying shape of the real density that is assumed to be smooth.

A classical way to release the empirical frequencies of a given data set from random variations is the use of parametric models that in a perfectly smooth way display the form of the underlying empirical density. However, the choice of a model that accurately describes the shape of a density is not always straightforward.

An innovative way to graduate an empirical frequency distribution in order to discover the smooth shape of the underlying density is a technique initially proposed by Rosenblatt (1956), Whittle (1958) ands Parzen (1962) with the odd name kernels. Kernel density estimation is a non-parametric way to simply obtain a graphical illustration of the shape of a distribution, without the imposition of a parametric model. It can be viewed as a formalization of a graphic approach. It can also be considered as a histogram in which the boxes have now been substituted by a smooth function, called kernel.

Considering a set of n independent and identically distributed random variables $X_1, X_2, \ldots, X_n$ with distribution function F and density function f, the kernel density estimator of f at the point x is given by the formula:

$$\hat{f}_K(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left[\frac{x - X_i}{h}\right],$$

where h is a smoothing parameter that represents the bandwidth around the point x, and K is the kernel function which, is in turn a probability density. Obviously, this estimator depends on the input data, the form of the kernel function used and the value of the bandwidth parameter chosen. As Silverman (1986) pointed out, this estimator can be considered as a sum of bumps placed at the observations, imposing a window of width h around the point x summarizing all the data points according to the kernel function used. The kernel function K determines the shape of the bumps, while the bandwidth h determines their width.

The kernel function K has the fundamental properties of a density function, namely:

$$K(x) \geq 0, \qquad \text{and} \qquad \int_{-\infty}^{\infty} K(x) = 1$$

The most widely used kernel functions are the Epanechnikov and the Gaussian or normal one. Silverman (1986) presents these two choices along with other alternatives, also providing an evaluation of their efficiency.

The bandwidth parameter h regulates the degree of smoothing. If h is small then nearby points are more influential. Alternatively, if a large bandwidth is used then information is averaged over a larger region, and as a consequence individual points have less influence on the estimate. Obviously, the choice of bandwidth is critical for the shape of the graduation provided. For the choice of the optimal bandwidth several methods have been proposed. Hardle (1990) pointed out that though effective data analysis has often been done by a subjective trial-and-error approach, the usefulness of kernel density estimation would be highly enhanced if an efficient objective method for the selection of an optimal bandwidth could be developed. Hence various data-driven methods for choosing the bandwidth have been proposed. All these methods have the common goal to determine an optimal bandwidth that minimizes the Mean Integrated Square Error (MISE),

$$E\int\{\hat{f}_K(x)-f(x)\}^2\,dx$$

The idea of finding an optimal bandwidth parameter in the sense of minimizing the MISE is introduced by Parzen (1962). Silverman (1986) provides an analytical review on the properties of such a smoothing parameter. Among the simplest proposals for the choice of the bandwidth parameter are various versions of the rule of thumb, according to which *f* belongs to a pre-specified class of density functions (Silverman, 1986; Hardle, 1991). In the case of a Gaussian density *f* when a Gaussian kernel function is used, the rule of thumb can either be based on the standard deviation or on the interquintile range. The resulting optimal bandwidth in the first case is given by the formula $h_{\mathrm{opt}}=1.06\hat{\sigma}\,\mathrm{n}^{-1/5}$. While in the case of the interquintile range, defined as $\hat{R}=X_{[0.75n]}\text{-}X_{[0.25n]}$, the optimal bandwidth is given by the formula: $h_{opt}=0.79\,\hat{R}\,n^{1/5}$. More accurate results can be obtained if the adaptive estimate of spread, that is the quantity $A=min$ (sample standard deviation, interquintile range/1.34) is used instead of σ in the formula for the optimal bandwidth.

A widely applicable method for choosing an adequate bandwidth is presented by Terrell (1990). This method makes use of the maximal smoothing principle that implies the idea of choosing the largest degree of smoothing that is compatible with the scale of the density of the estimates. The main advantage of this method is that it tends to eliminate accidental features such as asymmetries or random multiple modes.

Among the several alternative methods proposed for the calculation of the bandwidth parameter particular emphasis is given to the method of cross-validation. This method is popular since it provides in a simple way a bandwidth that reflects the special features of an empirical density also considering smoothness. Two forms of cross-validation are presented, maximum likelihood cross-validation and least-squares cross-validation. The least-squares cross-validation bandwidth selector proposed by Rudemo (1982) and Bowman (1984) is the most widely used. This was proven to give a bandwidth that converges to the optimal under very weak conditions (Stone, 1984). However, Hall and Marron (1987a), through many simulation studies and real data examples, are drawn to the conclusion that in many cases the performance of this method is rather disappointing since it suffers from a great deal of sample variability. Many alternative methods have been proposed in the literature for the selection of the bandwidth parameter. Among them the methods of the plug-in rules (Hardle, 1990) and the biased cross-validation (Scott and Terrell, 1987) are of special interest. Marron (1988) provided an improved version of the least-squares cross-validation, known as partitioned cross-validation. Various suggestions about the improvement of the cross-validation bandwidth selection regarding its properties in comparison with the optimal bandwidth selection are given in Davis (1981), Hall and Marron (1987b), Sheather and Jones (1991), Hall et al (1991), Fan and Marron (1992). Hardle (1990,1991) provides an analytical review on several methods for the choice of the bandwidth.

Although there are complex theoretical issues involved in the choice of choosing the bandwidth parameter, the various applications presented reflect an absence of automatic selection algorithms. Deaton (1989) uses the informal and arbitrary method of inspection, whereas Quah (1996) uses the method of a reference distribution, where the smoothing parameter would derive as the optimal bandwidth minimizing MISE if both data and kernel were Gaussian. Silverman (1986) points out that this method oversmooths multimodal and highly skewed densities because the width is too wide. Bowman and Azzalini (1997) accept that the assumption of normality is potentially a self-defeating one when attempting to estimate a density non-parametrically, but for unimodal distributions it gives at least a useful choice of smoothing parameter which requires very little calculation. They also mention that this approach to smoothing has the potential merit of being cautious and conservative. Since the normal is one of the smoothest possible distributions, the optimal value of the bandwidth parameter will be large. If this is then applied to non-normal data it will tend to induce over-smoothing. The consequent variance reduction has at least the merit of discouraging misinterpretation of features, which may in fact be due to sampling variation. However, this approach is considered to have the best results compared to those given by the spread estimated from the data or the interquintile range. Thus an estimate of the spread is used resulting in the following formula for the calculation of the bandwidth: $h=0.9*A/n^{1/5}$ where $A=min$ (sample standard deviation, interquintile range/1.34).

In practice, the use of fixed kernel estimates for comparing several densities requires special attention to the bandwidth parameter used. As Marron and Schmitz (1992) pointed out, since the fixed kernel estimate is dependent in the bandwidth parameter, accurate comparisons can be made only when the same amount of smoothing is applied to each curve. They, therefore, propose making use of the cross-validation technique for the derivation of the optimal bandwidth for each curve and then considering the average of the cross-validated bandwidths, in order to represent the same amount of smoothing and make all kind of comparisons between the different densities possible. This approach has also been adopted by Hildenbrand, et. al. (1998).

Although fixed kernel estimates enable us to reveal the shape of a distribution, they usually lead to misleading results regarding the tails of the distribution due to the sparseness of the data at these data points. A solution to this problem is the use of adaptive kernel estimates. Adaptive kernel estimates allow the smoothing parameter to be adapted for local efficiency in different parts of the distribution, which means that a smaller bandwidth is used in areas of high data densities while the value of the bandwidth increases in areas of low data densities. This adaptive procedure retains detail where observations concentrate and eliminates noise fluctuations where data are sparse. Silverman (1986) describes the adaptive kernel approach as a two-stage procedure, which relies on pilot estimates of the density obtained from fixed kernel estimates. So, as a first step, fixed kernel estimates permit us to obtain a general view of the shape of the distribution. Then a local window factor is calculated at each sample point given by the formula:

$$w_i = \left(\frac{\bar{f}_g}{f_K(x_i)} \right)^{\frac{1}{2}}$$

where $\bar{f}_g = \left(\prod_i \hat{f}_K(x_i) \right)^{1/2}$

These local window factors are used to adjust the bandwidths over the range of the data and consequently construct the adaptive kernel estimates. Thus the adaptive kernel estimate is given by:

$$\hat{f}_A(x) = \frac{1}{nh}\sum_{i=1}^{n}\frac{1}{w_i}K\left[\frac{x - X_i}{w_i h}\right]$$

In terms of the choice of the pilot estimate the Gaussian distribution can be used as a reference standard in choosing the bandwidth (Tukey, 1977; Scott, 1979; Silverman, 1986), resulting in $h=0.9*A/n^{1/5}$ where $A=min$ (sample standard deviation, interquintile range/1.34).

Recently, the use of non-parametric density estimates, making use of the kernel techniques in order to represent the shape of income distributions, has gained particular interest. Cowell, et al.(1994) and Jenkins (1995) suggest the use of kernel density estimates in order to reveal the features of the shape of the UK income distribution. Cowell, et al. uses both fixed and adaptive bandwidhts and the Epanechnikov as well as the Gaussian kernel functions. Jenkins (1995) also uses adaptive bandwidths. Schluter (1996) utilizes kernel density estimates for extracting the shape of income distribution in Germany. In his work the bandwidth used is selected by inspection. Beginning from a small bandwidth value he gradually increases it until he achieves smooth estimates. A non-parametric analysis of household income distribution for the population of Great Britain from the years 1968 to 1995 is also provided by Hildenbrand et al. (1998). Moreover Biewen (2000) considers kernel density estimates in order to study the shape of the income distribution of Germany. As Schluter (1996) mentions, in comparison with parametric models kernels avoid distributional assumptions and appear to be a natural method for exploratory analysis of income distributions (Schluter, 1996). Cowell, et al. (1996) mention that the use of kernel estimates is not prone to individual judgement as it happens in the case of more traditional approaches used, such as inequality and poverty indices and, therefore, the robustness of the results is enhanced. They also argue that the use of kernels permits the tracing of interesting details regarding the different patterns that cannot be revealed and depicted when using indices.

3 The Data

This study is based on cross-sectional data from the national household panels of Germany, Luxembourg, the United Kingdom (UK), Poland and Hungary. These datasets are part of the PACO (Panel Comparability) database that created in the frame of the PACO Project (Schaber et al, 1993).[1] The aim of this project is to introduce a centralised approach for developing an international comparable database of data from different national household panels. This database contains data in a micro-level referring to a wide range of household and personal characteristics and covering a large number of years. The variables included on the PACO database are harmonised in the sense that they have identical structures for the different countries. Common data and variable definitions are also adopted assisting the cross-national compatibility of the data.

In our analysis the annual gross household income for the year 1994 is used. This is the household income before direct taxes and social security contributions. In order to compare households with different size and composition, the OECD revised scale is

[1] PACO database has been created by the *Centre d'Études de Populations, de Pauvreté et de Politiques Socio-Économiques / International Networks for Studies in Technology, Environment, Alternatives, Development* (CEPS/INSTEAD), Luxembourg in partnership with the *Deutsches Institut für Wirtschaftsforschung* (DIW), Germany.

applied to the annual gross household incomes. According to this scale a weight of 1 is assigned to the first adult of the household, and a weigh of 0.5 and of 0.3 are assigned respectively to each additional adult and child. Since our focus is on cross-national comparison of the way that income is distributed, normalised income is considered. This is the annual equivalent household income divided by the corresponding mean equivalent income of the country. This allows us to have a more concrete view on inequality differences. The issues related to the differences on income distribution between population subgroups and to the impact they have on the overall inequality were investigated by providing kernel estimates of population subgroups formed according to the age (elderly and non elderly) and educational level of the head of household.

For our calculations, STATA programmes for kernel density estimation written by Salgado-Ugarte et al. (1993) were used. The adaptive kernel estimates for each dataset considered were calculated at 500 equally-spaced points derived from the corresponding normalised equivalent incomes. These incomes were truncated at values no greater than five times the country's mean. Adaptive kernel estimates were obtained as a result of a two-stage procedure. At an initial step fixed kernel estimates were obtained using an optimal bandwidth. The optimal bandwidth for each data set was calculated by the formula: $h=0.9*A/n^{1/5}$ where $A=min$ (sample standard deviation, interquintile range/1.34). Then, based on these estimates, local window factors were calculated through which the adaptive kernel estimates were obtained. In order to derive both the fixed and the adaptive kernel estimates the Gaussian kernel function,

$$K(t)=(1/\sqrt{2\pi})*e^{-(1/2)t^2}$$

was used. Although the Epanechnikov kernel function is theoretically considered to have the best behaviour, as providing minimum MISE, in practice the choice of the kernel function does not affect the final results.

4 Assessing Income Distribution Between Countries and Population Subgroups

A conventional approach of investigating inequality and distribution of income among and between countries is by using certain inequality indices and summary measures. In Table 1 we present estimates on inequality based on some of the most well known and widely used indices; the Coefficient of Variation (C), the Relative Mean Deviation (M), the Logarithmic Variance (V), the Gini (G) index, the Atkinson indices for ε=0.5 ($A_{(\varepsilon=0.5)}$) and ε=2 ($A_{(\varepsilon=2)}$), the Theil's Entropy index (T), the Mean Logarithmic Deviation (L), and the Half the Squared Coefficient of Variation ($C^2/2$).[2] The last three indices are part of the family of Generalised Entropy measures $E_{(\theta)}$: T is the $E_{(1)}$, L is the $E_{(0)}$, and $C^2/2$ is the $E_{(2)}$ (see Appendix 1). All the above indices have been extensively used by researchers in the field and allow for the (potential) comparison with the findings of other studies. In addition, these indices, with the exceptions of M and V, fulfil the most desirable properties; the anonymity, the mean independence, the population independence and the principle of transfers.

[2] A detailed presentation of inequality indices and their properties can be found in Atkinson 1983, Atkinson and Bourguignon (2000), Anand 1983, Jenkins 1991, Lambert 1993, Cowell 1995, Sen 1997. The definitions and formulas of the inequality indices used in the present study are presented in the Appendix.

As we can see in Table 1 any attempt to rank these countries according to the degree of inequality is affected significantly by the particular inequality index used. Thus, estimates of Gini index rank the UK first, as the country with the highest inequality followed by Poland and Germany which have almost identical values of Gini.[3] Hungary is in the fourth place while Luxembourg is the country with the lowest inequality.[4] This ordering changes when other inequality indices are used. Estimates based on the Coefficient of Variation (C) shows again that the UK is the country with the highest inequality followed by Poland. Hungary is now in the third place followed by Germany. However, the estimated value of the Coefficient of Variation shows that income inequality is quite similar for the last two countries. When our exercise is based on $A_{(\varepsilon=2)}$ we get quite a different picture concerning the inequality ordering between these countries. Poland is the country with the highest inequality followed by the UK, with Germany in the third place and Hungary in the forth. Similar differences are observed when this ordering is based on other inequality indices and summary measures. The only finding that all inequalities indices agree on is that Luxembourg is the country with the lowest inequality.[5] These differences in inequality ordering have also been observed and documented in similar inequality exercises (see Smeeding 1991, Atkinson et al 1995).

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[3] Indeed a number of studies in the field show that in the 1990's inequality in the UK, measured by the Gini index, is quite high among industrialized countries (Gottschalk and Smeeding 2000, Eurostat 2000, Joseph Rowntree Foundation 1995).

[4] Estimates on the Gini index provided by the World Bank (2000) also show that in the mid 1990's inequality was higher in Poland than in Hungary. However, similar figures concerning the inequality ordering between these two countries were also found in the mid 1980's (see Atkinson and Micklewright 1992). Of course, both countries have relatively low inequality compared to the other transition economies of Europe and Central Asia (World Bank 2000).

[5] As already proved, all inequality indices that satisfy the mean independence, the population independence and the principle of transfers will give an unambiguous ranking of various distributions, only when the corresponding Lorenz curves do not intersect (Anand 1983, Lambert 1993). Based on

Table 1. Aggregate inequality indices for Germany, UK, Luxembourg, Poland and Hungary.

INEQUALITY INDICES	Germany	UK	Luxembourg	Poland	Hungary
Coefficient of Variation (C)	***0.654***	***0.954***	***0.490***	***0.885***	***0.667***
Logarithmic Variance (V)	***0.547***	***1.750***	***0.193***	***0.627***	***0.352***
Relative Mean Deviation M	***0.459***	***0.733***	***0.340***	***0.452***	***0.436***
Gini (G)	***0.324***	***0.490***	***0.242***	***0.325***	***0.309***
Atkinson $A_{(\varepsilon=0.5)}$	***0.090***	***0.204***	***0.048***	***0.095***	***0.079***
Atkinson $A_{(\varepsilon=2)}$	***0.472***	***0.918***	***0.171***	***0.957***	***0.302***
Mean Logarithmic Deviation (L)	***0.205***	***0.535***	***0.096***	***0.213***	***0.163***
Theil (T)	***0.180***	***0.401***	***0.100***	***0.209***	***0.170***
Half the Squared Coefficient of Variation ($C^2/2$)	***0.214***	***0.455***	***0.120***	***0.391***	***0.223***

The estimates are based on household's equivalent disposable income.

As already noted, the above differences in inequality ordering are attributed to the fact that inequality indices are not value free.[6] Each of the above indices places more emphasis in transfers at particular parts of the distribution and implies a certain value judgments about society (see Atkinson 1983, Cowell 2000, 1995, Lambert 1993,

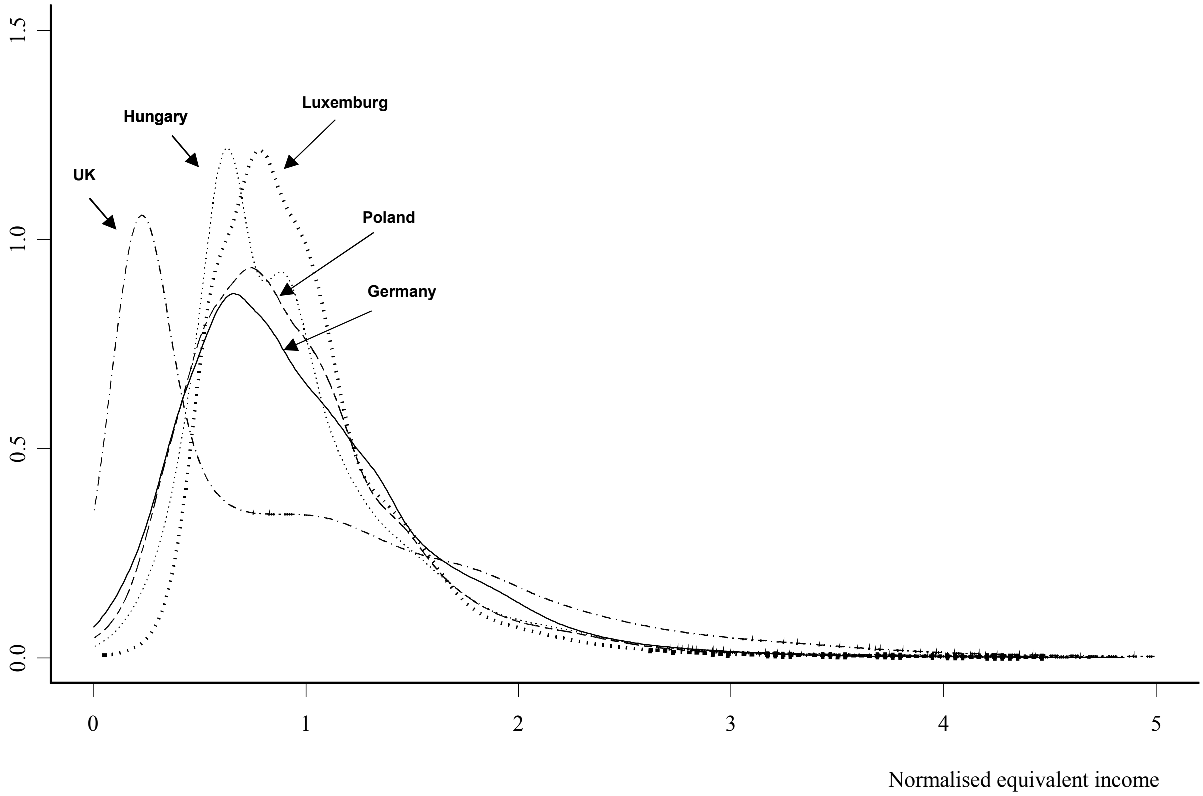
the present study's findings, we can hypothesise than the distribution of income in Luxembourg Lorenz-dominates the relevant distributions of the other countries.

[6] Inequality indices can be generally classified in two main categories: objective and normative (Sen 1997). Indices such as the C, the M and the G are considered objective measures since they are interested in the distribution of some particular attributes, such as income or earnings, using statistical measures. By contrast, indices such as those proposed by Atkinson are labeled as normative since they are based on particular social welfare functions. However, although the objective indices are not explicitly derived from any particular concept of social welfare, they introduce certain value judgments in assessing inequality since they weight differently transfers at various points of the income scale (Atkinson 1983, Lambert 1993, Sen 1997, 2000).

Jenkins 1991, Sen 1997). Thus G is more sensitive to differences at the middle of the distribution while C is more sensitive to changes at the top. Atkinson indices are generally more responsive to transfers at the lower part of the income distribution. However, the sensitivity of the Atkinson index depends on the value of ε. A higher value of the inequality aversion parameter ε makes the Atkinson index more responsive to changes at the bottom end of the distribution. Thus $A_{(\varepsilon=2)}$ is relatively more sensitive to differences at the bottom of the distribution than $A_{(\varepsilon=0.5)}$. Finally within Generalised Entropy family indices the higher the value of parameter θ the higher is the emphasis placed in transfers at the top of the distribution. Therefore L is more sensitive to differences at the bottom of the distribution and $C^2/2$ is more sensitive to differences at the top. Thus assessments made using certain inequality indices could lead to different views and conclusions about the inequality ordering between countries and may have a significant influence on design policy intervention at a national or international level. Of course, using a number of alternative indices could help us capture the different aspects of inequality and test the robustness of the findings. However, as Table 1 shows, it is hard for the non-expert to assess these differences in inequality for the above countries based solely on these finding. Even those who are familiar with the properties of these indices will face difficulties in trying to paint the complete picture concerning the exact shape and characteristics of income distribution for each country.

Kernel density estimates could prove a quite valuable tool in this inequality exercise. As noted above, kernel density functions provide a smooth representation of income distribution. Thus it could assist us in making the comparisons between different countries and population groups easier and more straightforward. Figure 1 presents the income density function for all countries.

Figure 1: Kernel estimates of the income distribution for the total population, 1994.



The income density function shows the concentration of population at each income level. The area under the curve between any pair of points at the income scale (horizontal axis) shows the proportion of the population with income between these two income levels.[7] A rule of thump for assessing these curves is that the higher the

[7] Of course one of the disadvantages of the above density functions is that it shows more clearly what it is happening at the low and middle income ranges than at the upper tail. In order for these diagrams to fit in the page, we restricted the horizontal axis to an income level equal to five times the country's mean. Thus some of the very large incomes are not presented. As noted by Cowell (1995), if in similar

concentration around the country's mean equivalent income is the lower the inequality will be. We make use of the words "mountain", and "hills" in order to describe curves' shape and consequently the population concentrations in various parts of the income scale. By "mountain", a metaphor also used by Jenkins (1996), we will refer to the highest population concentration on the income scale while by "hills" we will refer to other secondary concentrations and bumps on density function. At first glance, Luxemburg appeared to be the country with the lowest inequality. The mountain of the frequency density curve is the highest among all countries, and also the narrower one, with its peak close to the country's mean income. The vast majority of the Luxembourg households have income between ½ and 1½ of the country's average. It is also the country that has the lower concentration of population with income higher than the double of the country's average. These could justify why all inequality indices used in Table 1 agree that Luxembourg is the country with the lower inequality. Hungary appeared to have the second higher population concentration, but its peak corresponds to a lower income level than that of the relative curve for Luxembourg, as well as for Poland and Germany. However, Hungary's curve has a second hill (peak) at the right slope of the mountain, which corresponds to an income level closer to the country's mean. The kernel density function for Poland and Germany shows a similar pattern concerning the population concentration on income scale, with Poland's density mountain being relatively higher but narrower than Germany's. This could explain why inequality indices such as Gini, which are more responsive to the transfers at the middle of the distribution, give similar figures for Poland and Germany. By contrast, inequality indices which are more sensitive to transfers at the top of the distribution, such as the Coefficient of

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scale diagrams all the large incomes were represented, diagrams would have to be more than 100

Variation, give higher figures of inequality for Poland. Germany showed higher concentration that the other three countries for incomes higher than the 1½ of the country's average. However, the concentration along the income scale, for incomes higher than three times the country's mean, seems to be identical for all countries and it is difficult to observe differences.

The UK is the country in which the density function that characterises the income distribution has a very unique shape compared to that of the other countries. Although the curve's mountain is higher than the relevant for Germany, its peak corresponds to a point at the income scale which is lower than the half of the country's mean income. In other words, by adopting a poverty line equal to the half of the county's average equivalent income, the UK's density function is the only one that shows the highest population concentration at an income level lower to the poverty line. The UK's income distribution reduces less sharply than that of the other countries, and has a thicker right tail. Thus it is the country that shows the highest population concentration for income level higher to 1½ of the country's mean. These findings concerning the shape of the UK's frequency density diagram agree with that of other studies in the field (Jenkins 1996, Cowell et al 1996, Joseph Rowntree Foundation 1995). These studies showed the there was an increase in income inequality in the UK during the 1980s (see also Johnson and Webb 1993, Atkinson 1996).[8] The shape of income distribution between 1979 and 1991 suggested that during that period a significant shift in population concentration to the high incomes

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meters long.

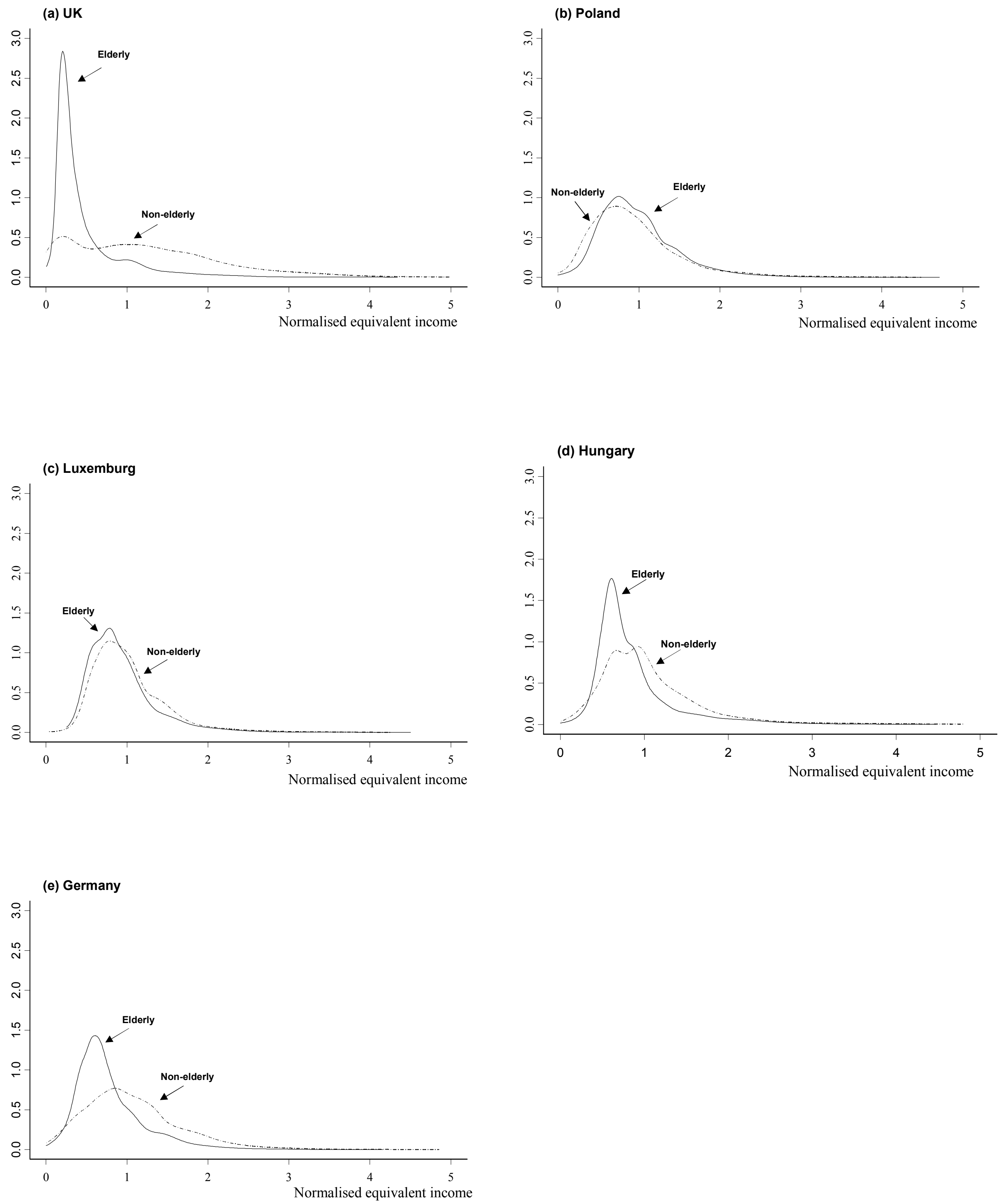
[8] Daly et al (1997), using kernel density estimates, found that inequality in Germany has also increased between 1984 and 1991. However, Schluter (1998), using stochastic kernels, found that during the 1980's and 1990's the intra-distributional mobility was higher in the US and the UK than in Germany. In fact, mobility in Germany was found to have changed only a little over this period.

had taken place and, at the same time, an increase in concentration at the particular low incomes (see Jenkins 1996, Cowel et al 1996). The income share of the 10% of the richest households increased significantly, combined with a fall in the income share of 20% of the poorest population (Joseph Rowntree Foundation 1995). Several factors, such as the increase of unemployment, the rising of self-employment, or the growth of investment income, have been considered responsible for this rise in the UK's income inequality. The increased polarisation between population subgroups - such as between those with and without occupational pensions, between those with high and low education, or between those in work and those without earnings – has been offered as an explanation for this unique shape of the UK's income distribution (see Johnson and Webb 1993, Atkinson 1996, Jenkins 1996, Joseph Rowntree Foundation 1995). Kernel density estimates could of course help investigate the polarisation between certain population subgroup and its impact on the distribution of income.

One factor that is generally acknowledged as having an important impact on household income is age. Sharp differences in income could be observed between elderly and non-elderly persons. Obviously, the income of non-elderly would be higher since it is attributed mainly to wages and salaries, which are generally higher than pensions. In addition, non-elderly income would show higher inequality since the dispersion of primary income among recipients is usually higher. Elderly income would be lower and more equally distributed since it is mainly attributed to state pensions and certain social security allowances. These income sources do not show the same high dispersion as primary income. Thus the elderly would have a highest concentration at an income level that corresponds to state pensions and other relevant allowances.

The importance of assessing the income differences between elderly and non-elderly households for policymakers is apparent. Relevant comparisons at a local, national and international level could provide significant information for effective policy interventions, particularly in the social policy area. Kernel density estimates could prove a useful tool for performing these comparisons between and within countries, concerning the differences in income distribution between the elderly and the non-elderly. Figure 2 shows the density functions for the elderly and the non-elderly population between these five countries. The definition of an elderly household was one with a head 65 years old and over. Luxembourg appears again to be the country with the fewer differences in the distribution of income between the elderly and the non-elderly. Both distributions show a high concentration close to the country's mean income. The elderly income appears more equally distributed since it shows a relatively higher population concentration than the non-elderly. However, the peak of the curve's mountain for the non-elderly is at a slightly higher income level. Similar small differences in the shape of the income distribution between the elderly and the non-elderly were found for Poland, although it is the only country in which the elderly group shows a population concentration at a slightly higher income level than the non-elderly. Hungary and Germany showed more significant differences concerning the distribution of income between the elderly and the non-elderly. The shape of the corresponding curves suggested that for both countries the income is again more equally distributed among the elderly than among the non-elderly. The curves'

Figure 2: Kernel estimates of the income distribution of the elderly and the non-elderly, 1994

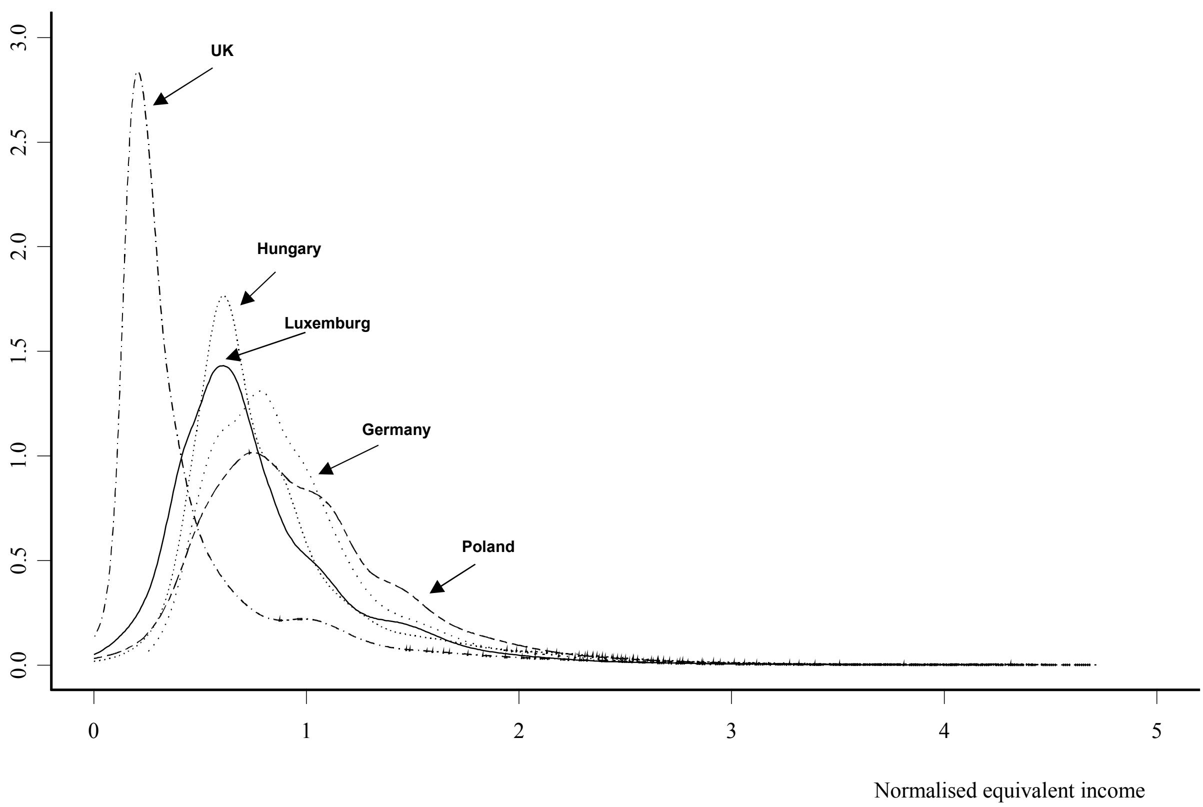


mountains are higher and narrower for the elderly group. However, the mountain's peak for the non-elderly group is closer to the country's mean income while for those in the elderly group it is at a considerable lower level. These findings suggest that the average income for the elderly in both countries is significantly lower than that for the non-elderly.

The sharper differences concerning the shape of the distribution of income between the elderly and the non-elderly population were found in the UK. The corresponding curve for the elderly has its peak at an income level lower to ½ of the country's average and then declines sharply. By contrast, the density function for the non-elderly is almost parallel to the horizontal axis for up to an income level equal to two times the country's average and then declines smoothly, but remains above the relevant curve for the elderly population. These findings suggest that there are extreme differences concerning the average household income and the inequality between the elderly and the non-elderly population in the UK. The income among the elderly population appeared much more equally distributed but also considerably lower than among the non-elderly population. Thus the high polarisation between the elderly and the non-elderly could help us understand the unique shape of UK's income distribution.

The distributions of income among the elderly population for these countries are presented in Figure 3. We could first notice that the UK is the country with the highest and narrowest density mountain, which may indicate that income inequality among the elderly is relatively low. However, the UK is the only country in which the highest population concentration for the elderly corresponds to an income level lower than the ½ of the country's average.[9] In other words, the elderly in the UK seem to have very low incomes, compare to the country's average, than the elderly in the other countries. By adopting again a poverty line equal to 50% of a country's equivalent mean income, the UK is the country with the highest proportion of elderly population in poverty. These results could be explained by the increasing gap between those elderly who have occupational pensions and those who do not (Joseph Rowntree Foundation 1995). These findings raise questions about the living standards of the elderly in the UK and make us more sceptical on the performance of the country's system of pensions and other related benefits.

Figure 3: Kernel estimates of the income distribution of the elderly, 1994.

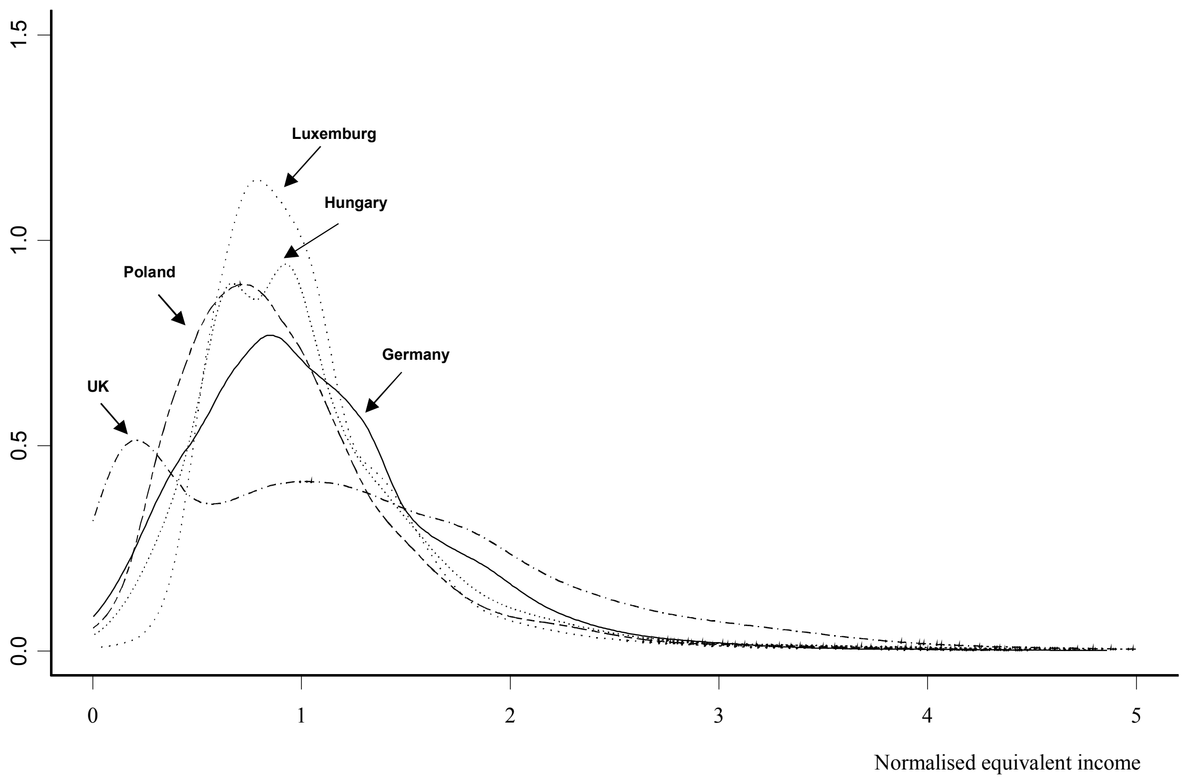


[9] Of course, only few people have incomes below this level that obviously corresponds to state retirement pensions and related benefits.

The relevant curves for Hungary and Germany have both a similar shape, with Hungary's density mountain being slightly higher and closer to the country's mean income. Luxembourg's density mountain has a similar shape and height with that of Germany. However, Luxembourg's mountain peak is at a higher income level and thus it is the country with the highest concentration of elderly at a level closer to the country's mean income. Poland's curve also shows a population concentration at a similar point of the income scale, but the density mountain is significantly lower than that for Luxembourg, which indicates a larger dispersion of incomes among the elderly.

Figure 4 presents the findings concerning the distribution of income among the non-elderly population between these five countries. Luxembourg appeared to be the country with the highest population concentration and the lowest dispersion of income among the non-elderly. The mountain's peak is close to the country's mean income. Hungary and Poland have both a similar shape of income distribution for the non-elderly population, but Hungary's curve indicates a population concentration at a slightly higher income level. In addition, Hungary's mountain has two peaks; one that corresponds to the country's mean income and one at a relatively lower income level. This could explain the shape of the distribution of income for total population, which shows another important peak at the right slope of the curve (Figure 1). The main peak of the curve is attributed to the join effect of the elderly population concentration and the concentration of the non-elderly that is represented by the first peak of the correspondent curve. The peak at the right slope of the curve for total population could be attributed to the non-elderly mountain's second peak. Germany's density function also shows a population concentration at the same point of the income scale as Luxembourg and Hungary. However, the Germany's density mountain is significantly lower and wider than that for Luxembourg and Hungary, which indicates a much larger dispersion of income. In addition, the right slope of Germany's curve is characterised my bumps and hills.

Figure 4: Kernel estimates of the income distribution of the non-elderly, 1994.



The relevant curve for the UK has again a very unique shape, which varies considerably from the typical uni-modal shape found in the other countries. The density function is almost parallel to the horizontal axis with some hills and bumps for up to an income level equal to two times the country's average. Then the curve declines smoothly, but remains well above the relative curves for the other countries. There is no significant population concentration (mountain) on the income scale. We can only observe two hills; one at a point lower to ½ of the country's mean income and the other at the level of the country's mean income. Cowell et al (1996) also found a similar bi-modal shape of income distribution for the UK's non-elderly population in 1988/89. The authors argued that this twin peak shape is attributed to certain changes that took place during the 1980's, which were characterised by an increase of the relative number of non-elderly who receive IS, combined with an increase of concentration of working population in higher income levels. [10] Therefore, the high polarisation between the elderly and the non-elderly could provide an explanation for the shape of income distribution for the total population (Figure 2). In particular, the concentration shows the corresponding curve for total population at an income level lower to ½ of country's mean is mainly attributed to the high concentration of elderly at that income level. Thus, we could assume that a significant part of the population concentration at this low income level is attributed to the high proportion of elderly receiving IS.

Education is generally acknowledged as an important factor in determining peoples' income. As a result, policy interventions in alleviating poverty and income inequality at a national and international level (ie EU) have put emphasis in certain educational reforms such as the introduction of compulsory and compensatory education and the

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[10] Cowell et al (1996) investigated further this unique shape of the UK's income distribution among the non-elderly by estimating the density functions for 1979 and 1988/89 for those non-elderly who receive Income Support (IS) and for those who do not receive it. The authors found that the two peaks in the distribution of non-elderly income correspond to the peaks of the relevant curves for IS-recipient and non-recipient. In addition, they found that between 1979 and 1988/98 there was a significant increase of the proportion of IS-recipients (Supplementary Benefits in 1979) and, at the same time, a shift of the mountain's peak for non-recipients toward higher income levels. According to Joseph Rowntree Foundation (1995), during this decade the population living in families with incomes IS increased by a

straightening of vocational training. The increased interest attracted by debates on social exclusion and marginalizing, especially within EU social policy, has given rise to the rhetoric which stresses the need for equal opportunities in training and education. It is, therefore, very important to have reliable and comparable information on the impact that the educational level has on the distribution of income in each country in order to be able to design and assess relevant national and international policy interventions. The density estimates of income distribution for population subgroups that were formed according to the level of education of the head of household are presented in Figure 5. Households in each country were divided in three distinguished groups according to the educational level of the head of household; *low-education* (those with up to primary education), *secondary-education* (those with upper or lower cycle secondary education), and *high-education* (those with college or university degrees). Luxembourg, Poland, Hungary and Germany show a similar pattern concerning the shape of the relevant density estimates of income distribution for these population groups. In all these countries the group of households with high education appeared to have lower and wider density mountains, while the relevant mountain peaks correspond to an income level higher than the country's average. Thus the households in which the head has a high-education are those with the highest average income but also with the largest inequality. Similarly, the households with low education have the highest and narrowest income frequency density mountains. The mountains' peak corresponds to a point of the income scale which lays between the ½ and 1 of the country's mean equivalent income. This clearly shows that the low educated are the population group with the significantly lower - but less unequally distributed - household income.

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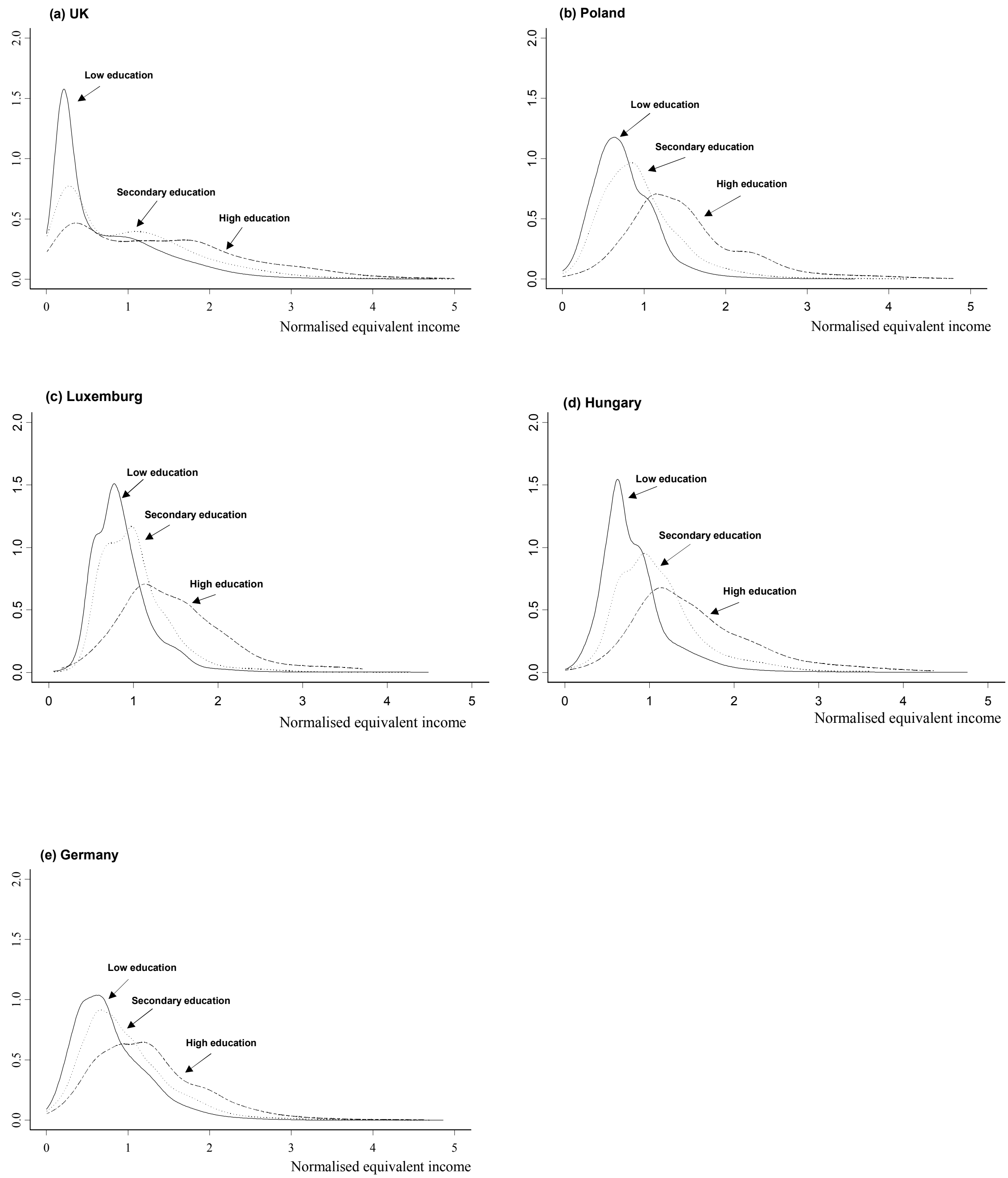
two-third, while those living at an income level lower to that of Income Support increased by a third.

Figure 5: Kernel estimates of the income distribution of households with low, secondary and high education, 1994.

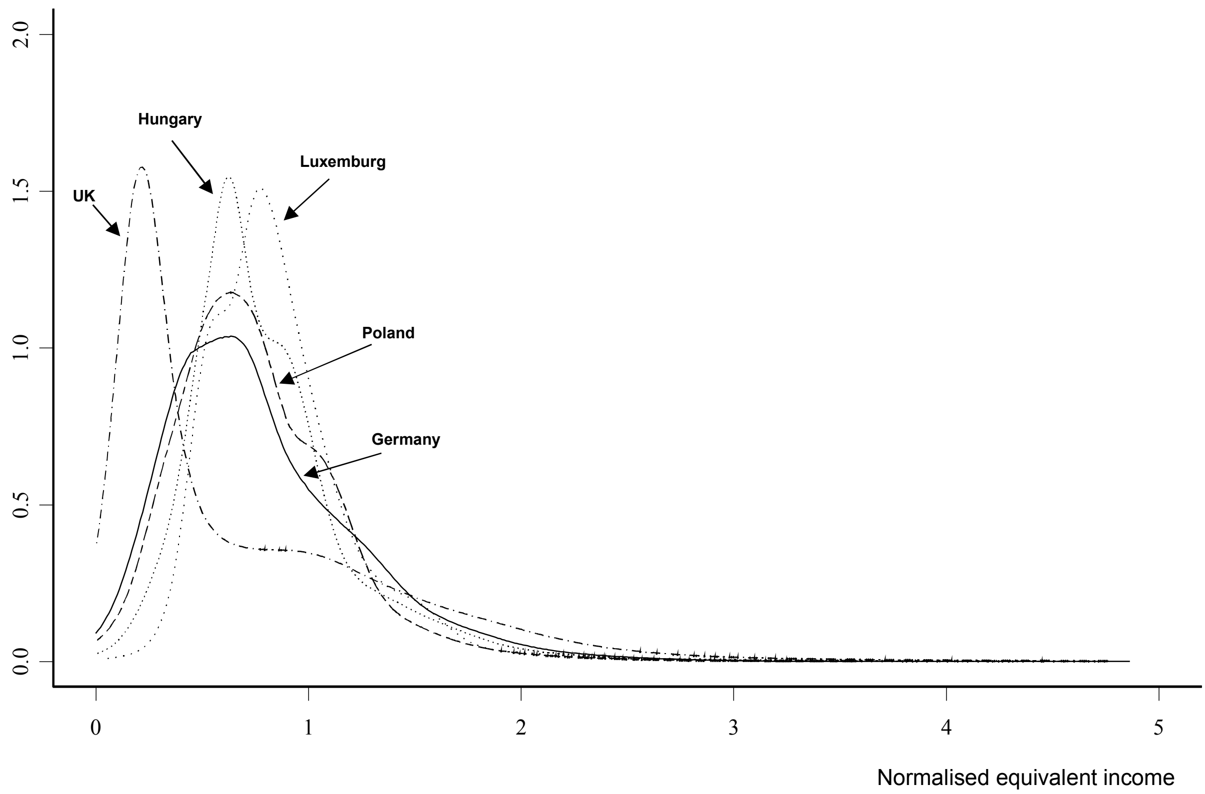


The shape of the relevant density function curves for those in secondary education is somewhere between the corresponding curves for the low and the high educated. In general, the shape of the curves indicates that those households with secondary education have an average income which is between the corresponding figure for those with low and those with high education. Similar comments could be made about the dispersion of income in this population group. However, in Poland and Luxembourg the distribution of income among those with secondary education has a more similar shape and pattern with the distribution of income of the low educated than with that of the high educated. This means that in these countries secondary education has either failed to provide people with the right qualifications, or the structure of the market (and the society) does not value this additional education in a way that would allow people with secondary education to gain rewards that differentiate them considerably from those with low education. By contrast, Hungary is the country in which the distribution of income among those with secondary education has a shape which is somehow closer to that of those with high education. In addition, Hungary is the only country where the mountain's peak for this group corresponds to the country's mean income. Germany on the other side is the country that shows the less sharp differences in the shape of income distribution between these population subgroups. The corresponding density functions of income distribution in the UK appear again to have a very unique shape. The three curves show to have a small concentration at the same point of the income scale that is well bellow the ½ of country's mean income. Following Cowell's et al's (1996) findings, we could suggest that this point corresponds to those people receiving IS. However, the height of the curves' mountain has a negative association with the level of education; the higher the educational level the lower the population concentration at this income level. Therefore, we could assume that the proportion of those receiving IS is negatively associated with the educational level. The relevant curves for those with secondary and high education are almost parallel to the horizontal axis for up to an income level that is equal to two times the country's mean and after that they decline smoothly. This indicates a high-income dispersion. A number of bumps and hills are observed in various points of the income scale.

The differences in the distribution of income between these countries for those with low education are presented in Figure 6. In general, the corresponding density curves for this population group follow a similar shape and pattern as those presented in Figure 1, which concern the distribution of income for total population. As we can see, the low educated in Luxembourg are those who have a quite high distribution density mountain that has its peak closer to the country's mean equivalent income. It is obviously the country with the lower inequality among this population group. Hungary's curve has a similar shape and height but its peak corresponds to a lower income level. In addition, Hungary's distribution shows a bump at the right slope of the curve while Luxembourg has a similar bump at the left slope. The relevant curves for Poland and Germany have a similar shape and mountain peak to that of Hungary's distribution. However, in these countries the population concentration at this point is less significant than that for Luxembourg and Hungary. Overall, Germany is the country with the shorter density mountain followed by Poland. The UK's income distribution for those with low education, as already noted, has a very different shape. It shows a high concentration at an income level lower to the ½ of the country's average income. The corresponding density mountain is even higher than those for Hungary and Luxembourg. Afterwards there is a sharp decrease for up to the level of

½ of average income and then the curve declines very smoothly, with some bumps, but remains above the relevant curves for the other countries. This is indicative of the high proportion of the low educated population in the UK that receive IS, and of the high dispersion on incomes among this group. In addition, it also raises questions concerning the efficacy of IS benefits to keep those in need at a decent standard of living, compared to the country's average standard.

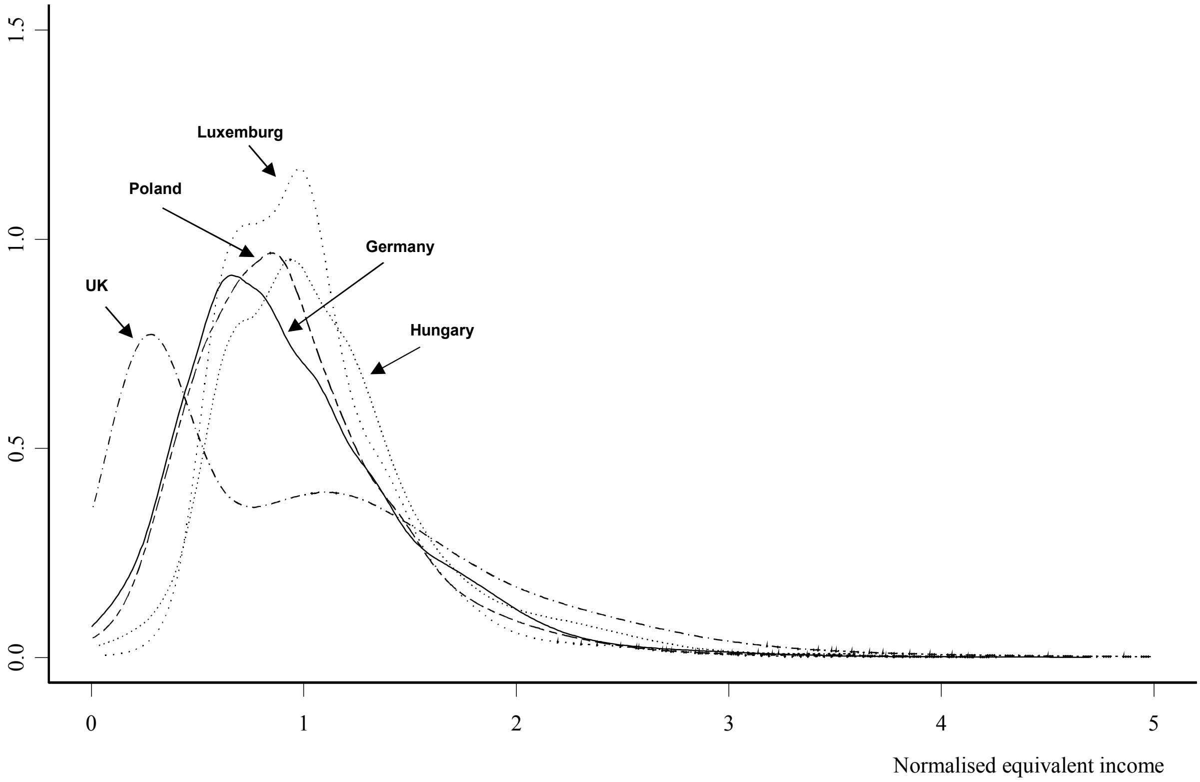
Figure 6: Kernel estimates of the income distribution of low educated households, 1994.



The distribution of income among those with secondary education, as Figure 7 shows, presents a different picture than the one for those with low education. Luxembourg is still the country that has the higher density mountain for this population group. German, Poland and Hungary show a relatively lower population concentration.

However the mountain's peak for Hungary corresponds to the country's mean income while for the rest of these countries the relevant peaks correspond to a relatively lower level of the income scale. As noted above, this is indicative of the way that each society (or market) values the qualifications attributed to secondary education. Thus Hungary is the only country that shows a high population concentration at the country's mean income for those with secondary education.

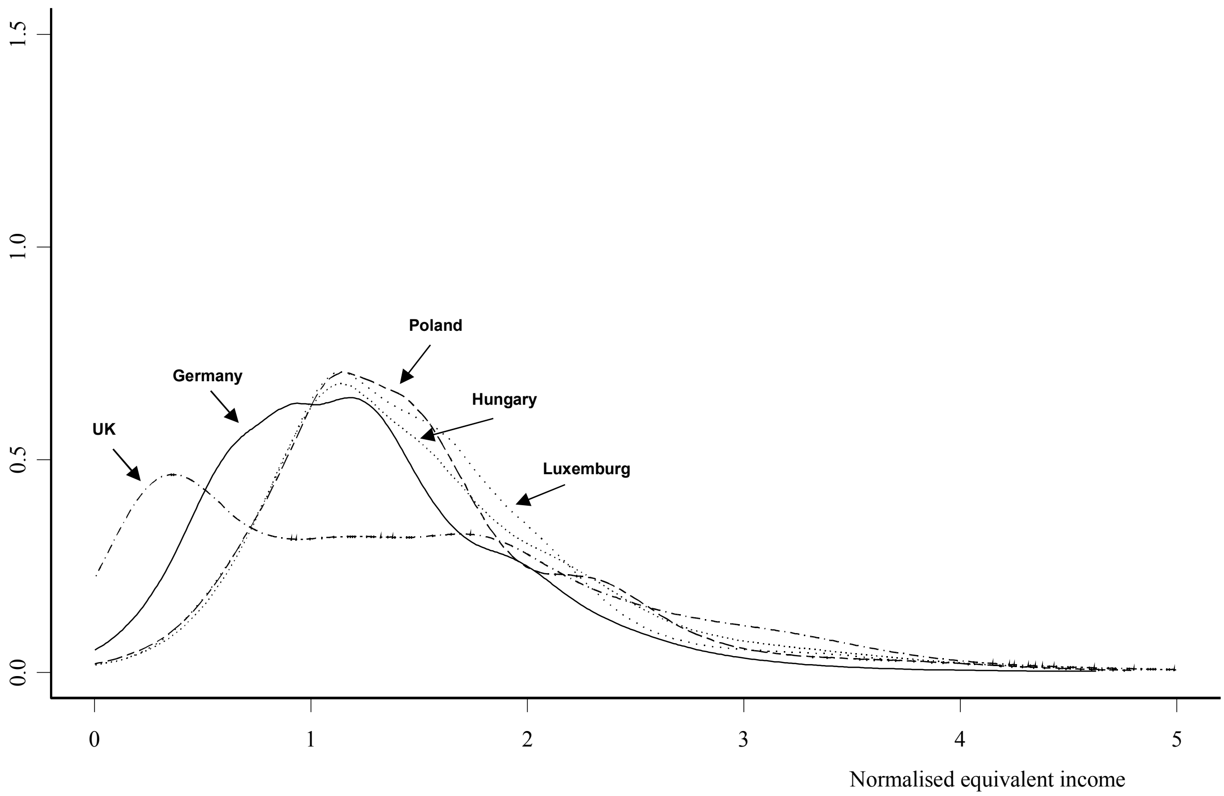
Figure 7: Kernel estimates of the income distribution among households with secondary education, 1994.



The corresponding density curve for the UK has again a very different shape. There is a high population concentration at an income level lower to the ½ of the country's mean. However, the UK's density mountain is the shorter among all countries. Just in the right slope of the density mountain there is also a hill, the peak of which corresponds to the country's mean income. This reflects an additional significant population concentration at this income level. Then the curve declines very smoothly, but with some bumps, and remains again above the relevant curves of the other countries. This is indicative of the high dispersion of income among those with secondary education in the UK.

The income distribution among those with high education, as presented in Figure 8, is very similar for Hungary, Poland and Luxembourg. The shape and size of the relevant density mountains are quite the same and their peaks correspond to an income level between the 1 and 1½ of each country's mean. However, differences can be observed in the curves' right slope and tails, which are characterised by a plethora of bumps. This probably indicates a high polarisation in different parts of the income scale, among those with high education in these countries.[11] The relevant curve for Germany has a similar shape and height with that of these three countries, but it indicates a population concentration at a relatively lower income level. In addition, in Germany the right slope and tail of this curve appeared smoother. Finally the UK's income density estimates for this population group present again a very different picture. The curve does not have the typical uni-modal shape. There is a small hill in the curve that corresponds to an income level lower to ½ of the country's mean. This, as we have already discussed, reflects the part of the population in this group which are IS receivers. After that the curve remains almost parallel for up to 2 times the country's mean income and then it is reduced smoothly but with some important bumps and hills on the right tail. This indicates a high income dispersion and polarisation among those with high education in the UK.

Figure 8: Kernel estimates of the income distribution of high educated households, 1994.



5 Conclusions

In this inequality exercise kernel density estimates were used in order to compare inequality and investigate the shape of income distribution between five European countries. Kernel density estimates proved to be quite revealing in mapping the differences in the distribution of income between countries in a simple and

[11] An analysis by occupational categories could illuminate this phenomenon.

straightforward way. These estimates also helped us explain the differences in inequality ordering that were found when a number of alternative indices and summary measures were used. The findings showed that income inequality is generally lower in Luxembourg, which proved to be the country with the highest concentration of population around the county's mean income. An unexpected finding was that economies in transition do not appear to have an overall income distribution pattern that differentiates them from the rest of the countries. By contrast, similarities in the shape of income distribution were found for Poland and Germany and for Luxembourg and Hungary (although Hungary showed a population concentration at a relatively lower income level than Luxembourg). The income distribution in the UK was found to have a unique shape compared to that of the other countries. It is the only country that shows the highest population concentration at an income level lower to ½ of the country's mean. It also appeared to have the highest population concentration at incomes higher to 1½ of the mean, which is indicative of the high overall dispersion of income.

Kernel estimates also helped us investigate and explain issues related to the impact that population concentration and income polarisation in various subgroups have on the overall inequality. Of course, similarities and differences were also found between countries in the way that income is distributed in various population subgroups. The distribution of income among the elderly and the non-elderly appeared to have an almost identical shape for Poland and Luxembourg. Differences between the elderly and the non-elderly were observed for Hungary and Germany where the corresponding curves in both countries suggested that the income is more equally distributed in the elderly population. The country that showed the sharper differences in the distribution of income between these subgroups was the UK. The findings suggested that there are extreme differences in the mean income and the inequality between the elderly and the non-elderly population in the UK. These differences could help us explain part of the main features of the income distribution in the total population. These finding are also questioning the effectiveness of the UK's social security system to preserve a descent standard of living for the elderly, compare to the county's average.

Similar differences in the distribution of income were observed between subgroups formed according to the educational level of the head of household. Luxembourg, Poland, Hungary and Germany showed similar patterns concerning the way that income distribution varies between educational groups. Generally, the level of education was found to be positively associated with the level of income and the degree of inequality. However, the differences in the shape of income distribution between educational levels were also found to vary between countries. This reflects the differences in the way the social structure and institutions of each society value the qualifications obtained in each educational level. Thus the average income and the income distribution among those with secondary education appeared to have more similarities with the group of low educated in Luxembourg and Poland and with the group of high educated in Hungary. Germany generally showed the least sharp differences in the shape of income distributions between these subgroups. The UK was the country in which the corresponding distributions have a unique shape and indicate extreme differences in population concentrations and polarisation between these population subgroups. All these subgroups in the UK showed a population concentration at an income level lower to ½ of the country's mean. Based on the findings of other studies, we could assume that this population concentration corresponds to those receiving IS. However, this concentration appeared to be negatively associated with the level of education. These findings could question the efficacy of the market institutions and the social policy in the UK to help a significant part of the population to maintain a decent standard of living.

The study's aim was to shed more light on the differences in income inequality between countries and thus to contribute to the knowledge needed for effective policy interventions (and evaluations) in tackling poverty and inequality. It opens the prospects for further research into this issue based on detailed information on the impact that population concentration and income polarisation in various subgroups have on overall inequality. Further research is obviously needed in order to improve our knowledge on the impact that certain social structures and main institutions, such as labour market, social security system and welfare regime, have on the way that incomes are distributed.

Appendix

The following definitions and formulas of the inequality indices and summary measures were used in the present study (see Atkinson 1983, Anand 1983, Jenkins 1991, Lambert 1993, Cowell 1995, 2000, Sen 1997):

Coefficient of Variation: $$C = \frac{1}{\mu}\sqrt{\frac{1}{\eta}\sum_{i=1}^{n}(\mu - y_i)^2}$$

were n is the population size, y_i is the equivalent income of the unit (household or person) i and μ is the mean equivalent income.

Relative Mean Deviation: $$M = \frac{1}{\eta}\sum_{i=1}^{n}\frac{|\mu - y_i|}{\mu}$$

Logarithmic Variance: $$V = \frac{1}{\eta}\sum_{i=1}^{n}\left(\ln\left(\frac{y_i}{\mu}\right)\right)^2$$

Gini index: $$G = \frac{1}{2\eta^2\mu}\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|$$

Atkinson family indices: $$A_\varepsilon = 1 - \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$ for $\varepsilon \geq 0$ and $\varepsilon \neq 1$

were ε is the inequality aversion parameter.

Generalised Entropy family indices for values θ other than 0 and 1:

$$E_\theta = \frac{1}{\theta(1-\theta)}\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{\theta} - 1\right)$$

while for θ=0 and θ=1 the Generalised Entropy index can be expressed in the forms:

$$E_1 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\mu}\ln\left(\frac{y_i}{\mu}\right)$$

$$E_0 = \frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{\mu}{y_i}\right)$$

were $E_1 = T$ (Theil's Entropy index) and $E_1 = C^2/2$ (Half the Squared Coefficient of Variation).

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