A) ΟΜΟΓΕΝΕΙΣ ΔΕΜΠ

$F(D_x,D_y)z=(D_x-\omega_1 D_y)(D_x-\omega_2 D_y)...(D_x-\omega_n D_y)z=0$.

$$\boxed{z=\varphi_1(y+\omega_1 x)+\varphi_2(y+\omega_2 x)+...+\varphi_n(y+\omega_n x)}.$$

$$\boxed{z=\varphi_1(y+\omega_1 x)+x\varphi_2(y+\omega_1 x)+...+x^{k-1}\varphi_k(y+\omega_1 x)+\varphi_{k+1}(y+\omega_{k+1}x)+...+\varphi_n(y+\omega_n x).}$$

$$\boxed{\varphi_1\left[y+(a+i\beta)x\right]+\varphi_1\left[y+(a-i\beta)x\right]++i\left\{\varphi_2\left[y+(a+i\beta)x\right]-\varphi_2\left[y+(a-i\beta)x\right]\right\}.}$$

A1) ΑΝΑΛΥΣΙΜΕΣ ΔΕΜΠ

$F(D_x,D_y)z=(a_1D_x+\beta_1D_y+\gamma_1)\cdot(a_2D_x+\beta_2D_y+\gamma_2)...(a_nD_x+\beta_nD_y+\gamma_n)z=0$.

$$\boxed{z=e^{-\frac{\gamma_i}{a_i}x}\varphi(a_i y-\beta_i x)\text{, αν } a_i\neq 0\text{, ή } z=e^{-\frac{\gamma_i}{\beta_i}y}\varphi(a_i y-\beta_i x)\text{, αν } \beta_i\neq 0.}$$

$$\boxed{e^{-\frac{\gamma_1}{a_1}x}\left[\varphi_1(a_1y-\beta_1x)+x\varphi_2(a_1y-\beta_1x)+...+x^{k-1}\varphi_k(a_1y-\beta_1x)\right]}$$

A2) ΜΗ ΑΝΑΛΥΣΙΜΕΣ ΔΕΜΠ

$$\boxed{z=\sum_{i=1}^{\infty}c_i e^{a_i x+\beta_i y}}$$

B) ΜΗ ΟΜΟΓΕΝΕΙΣ ΔΕΜΠ

$$F(D_x,D_y)z=(D_x-\omega_1D_y)...(D_x-\omega_nD_y)z=f(x,y)\Rightarrow\begin{cases}(D_x-\omega_nD_y)u_1=f(x,y)\\(D_x-\omega_{n-1}D_y)u_2=u_1\\.....\\(D_x-\omega_1D_y)z=u_{n-1}\end{cases}$$

$\left(D_x-\omega D_y\right)u=f(x,y)$. $\boxed{u=\int f(x,a-\omega x)dx}$. (μερική λύση μη ομογενούς)

$a\dfrac{\partial z}{\partial x}+\beta\dfrac{\partial z}{\partial y}=g(x,y,z)$. $\boxed{\dfrac{dx}{a}=\dfrac{dy}{\beta}=\dfrac{dz}{g(x,y,z)}}$. (προσαρτημένο σύστημα)

$\dfrac{du(x)}{dx}+p(x)u(x)=q(x)$. $\boxed{u(x)=e^{-\int p(x)dx}\left[c+\int q(x)e^{\int p(x)dx}dx\right]}$. (Γραμ. ΔΕ $1^{ης}$ τάξης)

1) ΚΥΜΑΤΙΚΗ ΕΞΙΣΩΣΗ

$$u(x,t)=\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi c}{l}t+b_n\sin\frac{n\pi c}{l}t\right)\sin\frac{n\pi x}{l}$$ όπου

$$a_n=\frac{2}{l}\int_0^l g_1(x)\sin\frac{n\pi x}{l}dx$$ και $$b_n=\frac{2}{n\pi c}\int_0^l g_2(x)\sin\frac{n\pi x}{l}dx.$$

2) ΕΞΙΣΩΣΗ ΤΗΣ ΘΕΡΜΟΤΗΤΑΣ

$$u(x,t)=\sum_{n=1}^{\infty}c_n e^{-\frac{n^2\pi^2 c}{l^2}t}\sin\frac{n\pi x}{l}$$ όπου $$c_n=\frac{2}{l}\int_0^l g(x)\sin\frac{n\pi x}{l}dx.$$

3) ΕΞΙΣΩΣΗ ΤΟΥ LAPLACE

$$u(x,y)=\sum_{n=1}^{\infty}\left(a_n\cosh\frac{n\pi y}{l}+b_n\sinh\frac{n\pi y}{l}\right)\sin\frac{n\pi x}{l}$$ όπου

$$a_n=\frac{2}{l}\int_0^l g_1(x)\sin\frac{n\pi x}{l}dx$$ και

$$b_n=\frac{1}{\sinh\frac{n\pi m}{l}}\left[\frac{2}{l}\int_0^l g_2(x)\sin\frac{n\pi x}{l}dx-\left(\cosh\frac{n\pi m}{l}\right)\frac{2}{l}\int_0^l g_1(x)\sin\frac{n\pi x}{l}dx\right].$$

$$u(x,y)=\sum_{n=1}^{\infty}\left(a_n\cosh\frac{n\pi x}{m}+b_n\sinh\frac{n\pi x}{m}\right)\sin\frac{n\pi y}{m}$$ όπου

$$a_n=\frac{2}{m}\int_0^m g_3(y)\sin\frac{n\pi y}{m}dy$$ και

$$b_n=\frac{1}{\sinh\frac{n\pi l}{m}}\left[\frac{2}{m}\int_0^m g_4(y)\sin\frac{n\pi y}{m}dy-\left(\cosh\frac{n\pi l}{m}\right)\frac{2}{m}\int_0^m g_3(y)\sin\frac{n\pi y}{m}dy\right].$$

$$v(x,t)=u(x,t)+\frac{x-l}{l}A-\frac{x}{l}B$$