

# Planimetry reliability with GPS

Vasileios K. Drosos\*<sup>1</sup>, Chrisovaladis Malesios<sup>2</sup> and Konstantinos P. Soutsas<sup>1</sup>

<sup>1</sup>Department of Forestry and Management of Environment and Natural Resources Democritus University of Thrace Ath. Pantazidou 193 Str., GR-68200 Orestiada, Greece vdrosos@fmenr.duth.gr

> <sup>2</sup>Department of Agricultural Development Democritus University of Thrace Ath. Pantazidou 193 Str., GR-68200 Orestiada, Greece <u>malesios@agro.duth.gr</u>

#### Abstract:

The first estimate of the forest surface area is often necessary to make appraisals directly to the size of the impact of an event such as fire or trespassing. Using known methods of surveying and photogrammetry requires time and money, while satellite ones give a general geographic information, while they have not confirmed a good accuracy. The use of different types of GPS from a relatively low cost (GPS handheld) to more expensive differential DGPS (two receivers and one, two or three frequencies) leads to sufficient accuracy to capture a few meters to a few centimeters. The easy to use of low cost GPS has given users euphoric inherent risks in the accuracy assessment of planimetry, particularly with low-cost GPS. The aim of this paper is to investigate the reliability of using different types of GPS in order to calculate the area of forest plots.

Keywords: reliability, planimetry, GPS.

# **1** Introduction

The first estimate of the forest surface area is often necessary to make appraisals directly to the size of the impact of an event such as fire or trespassing. Using known methods of surveying and photogrammetry requires time and money, while satellite ones give a general geographic information, while they have not confirmed a good accuracy. The use of different types of GPS from a relatively low cost (GPS handheld) to more expensive differential DGPS (two receivers and one, two or three frequencies) leads to sufficient accuracy to capture a few meters to a few centimeters. The easy to use of low cost GPS has given users euphoric inherent risks in the accuracy assessment of planimetry, particularly with low-cost GPS. What is the transmission of position error of surveying in the calculation of the area? The aim of this paper is to investigate the reliability of using different types of GPS in order to calculate the area of forest plots.

The Global Positioning System (GPS) is a satellite-based navigational system designed and operated by the US Department of Defense for military and civilian use.

Besides the standard use of navigation, GPS can be also extremely useful in other tasks, for instance in mapping forested areas, such as streams and forest roads, since that mapping by the utilization of a GPS receiver can significantly reduce positioning errors which are inevitable when measuring with conventional instruments, such as for instance the tape measure. Moreover, GPS is until now the only possible option in terms of cost and labor when mapping forests on a large scale (Tachiki et al. 2005).

The objective of this study is an attempt to examine and compare the performance of two GPS receivers of different orientation, one recreational and another more precise.



Accuracy and precision are often used to describe how good is the position that acquired by the GPS receiver under study. Accuracy is the degree of closeness of an estimate to its true, but unknown value and the precision is the degree of closeness of observations to their means. There is a series of accuracy and precision measures that have been developed.

The more common terms used in previous works to estimate GPS accuracy and precision are Circular Error Probable (CEP), Root Mean Square error (RMS) and Distance Root Mean Square error (DRMS). Sawaguchi et al. (2003) define CEP as the value witch a half of the data points fall within a circle of this radius centered on truth and a half lie outside this circle and use CEP to estimate GPS positioning a different forest type, antenna height, and season, and to clarify the relationship between sampling number and the convergence of positioning precision. RMS value mean that approximately 68 percent of the data points occur within this distance of truth. Yoshimura and Hasegawa (2003) use RMS testing on horizontal and vertical positional errors of GPS positioning at different points in forested areas. DRMS should be expressed clearly whether the accuracy value refers only to horizontal or to both horizontal and vertical and indicates that approximately 95 percent of the data points occur with this distance of truth (Dana 1999). It is the method proposed to calculate accuracy in the Standard Positioning Service (SPS) (Kaplan 1996). Dana (1999) defines 2DRMS as Estimated Positional Error (EPE) and is used to compare differences between GPS receiver under forest canopies (Karsky et al. 2000).

There are techniques as differential global positioning system (DGPS) that improve precision and accuracy under tree canopies. Hasegawa and Yoshimura (2003) achieved a mean error of a 1 to 30-min observation varied between 0.029-0.226 m (without closed tree canopies) and it was 0.415-0.894 m (with closed tree canopies), using Dual-frequency GPS receivers by carrier phase DGPS static surveying. Sawaguchi and others (2003) using DGPS got mean CEP95=2.80 m for deciduous broadleaved trees and 4.99 m for conifers. Additionally they demonstrated that positioning precision was not noticeably improved if the sampling number was around ten. So DGPS improve GPS positioning in precision, accuracy and efficiency because the observation time is shorter (Næsset 2001; Næsset and Jonmeister 2002).

# 2 Materials and Methods

# 2.1 Study areas

As for study areas were chosen three places with different land use like Phinikas a place on the skirts of the town planning complex of Thessaloniki which is forest botanical garden, Exohi a rural forest area nearby the suburban forest of Thessaloniki and last Taxiarchis a strictly forest area.

# 2.2 Accuracy Measures

Accuracy and precision are often used to describe how good GPS receiver acquires the position. Accuracy is the degree of closeness of an estimate to its true value. Reporting accuracy typically consists of summary statistics derived from ground measurements, Total Station measurements in our case.

For a digital elevation model, the statistic might be a Distance Root Mean Square error (DRMS) for a set of locations at which the true elevation is known (American Society of Civil Engineers 1983; American Society of Photogrammetry 1985; Shearer 1990; Goodchild 1991).

If a GPS receiver displays position coordinates that are different from the "true coordinates" of the antenna position, this is position error1. A vast variety of measures have been employed for measuring this error, i.e. the degree of conformance between the estimated or measured position.

<sup>&</sup>lt;sup>1</sup> Specifically, for each computed position i (i=1,2,...,n), the positional error, say  $D_i$ , is calculated as the deviation between the satellite obtained coordinate from the GPS receiver and the true reference coordinate from total station.



The more common terms used in previous works to estimate GPS accuracy and precision is Circular Error Probable (CEP), Root Mean Square error (RMS) and Distance Root Mean Square error (DRMS).

More specifically, as concerns the evaluation of the horizontal positional errors, we can distinguish among these measures the Distance Root Mean Square (DRMS), which is defined as:

$$DRMS(m) = \sqrt{\sigma_x^2(m) + \sigma_y^2}(m)$$
(1)

where  $\sigma x$  (m) and  $\sigma y$  (m) denotes the standard deviation of the positional error along the x axis and y axis, respectively, for matrix as size m and are calculated by the following formulas:

$$\sigma_{x}(m) = \sqrt{\frac{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}{n - 1}}$$
(2)

$$\sigma_{y}(m) = \sqrt{\frac{\sum_{i=1}^{n} (y_{i} - \overline{y})^{2}}{n - 1}}$$
(3)

where xi and yi the observed coordinates x (m) and y (m) the arithmetic means of the observed values, respectively, for matrix as size m.

Other horizontal position precision measures include the 2DRMS, which is twice the Distance Root Mean Square, the Circular Error Probability (CEP), which is the radius of circle centered at the true position, containing the position estimate with probability of 50%, is given by:

$$CEP(m) = 0.62\sigma_{v}(m) + 0.56\sigma_{x}(m)$$
(4)

The radius of the 95% is often quoted and the term R95 used. R95 is CEP with the radius of the 95% probability circle, calculated by the following formula:

$$R95(m) = R(m) \times \left[ 0.62\sigma_{y}(m) + 0.56\sigma_{x}(m) \right]$$
(5)

with R=2.08 when  $\sigma y / \sigma x=1$ . The latter two-dimensional precision measures can be easily extended in the three-dimensional space. Thus, Spherical Error Probable (SEP) applies to combined horizontal and vertical precision, given by:

$$SEP(m) = 0.51 \times \left[\sigma_x^2(m) + \sigma_y^2(m) + \sigma_z^2(m)\right]$$
(6)

corresponds to the CEP in the two dimensions, the Mean Radial Spherical Error (MRSE) is the 2D analogue of the Distance Root Mean Square:

$$MRSE(m) = \sqrt{\sigma_x^2(m) + \sigma_y^2(m) + \sigma_z^2(m)}$$
(7)

The mean positional error  $(\overline{D})$  was calculated from:

$$\overline{D}(m) = \frac{1}{n} \sum_{i=1}^{n} D_i(m)$$
(8)



The standard deviation (SD) of the positional errors Di was computed using:

$$SD(m) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left( D_i - \overline{D} \right) m}$$
(9)

In order to determine the area of a every polygon as with the total station as with the different types of GPS (DGPS and GPS-GIS) we are going to use the shoelace formula, or shoelace algorithm which is also known as Gauss' area formula, from Johann Carl Friedrich Gauss (30 April 1777 – 23 February 1855) who was a German mathematician and scientist who contributed significantly to many fields, including number theory, statistics, analysis, differential geometry, geodesy, geophysics, electrostatics, astronomy, optics also it has applications in surveying and forestry among the other areas.

$$2E(m^2) = \sum_{i=1}^{n} y_i \times (x_{i+1} - x_{i-1})$$
(10)

where E (m2) is the area of the polygon, (xi, yi) are the coordinates for every corner of the polygon, i = 1, ..., n are the vertices (or "corners") of the polygon, and xn+1 = x1.

Measurements with DGPS performed in Real time kinematic method. The rover was always nearby the base and with good satellite geometry (DDOP).

First we need to calculate positional errors along the x-and the y-axis, i.e. sx (m) and sy (m) for each area and for each GPS type separately. The calculated errors are shown in the following Table 10, along with the corresponding measure of Distance Root Mean Square (DRMS) = sp.

$$s_{p} = (s_{x}^{2} + s_{y}^{2})^{0.5}$$
(11)

This presupposes that there are not systematic errors of dependence between the coordinates and correlation between adjacent points.

According to researches (Reskik 2002) during the static method the correlation between measurements is very small (near 0), while at the kinematic method between 0.5 and 0.9. In the second case we have and systematic errors. In order to avoid large estimation errors we must have average distance measurement (polygon sides) and intermediate points' dependency, which is not always possible.

If the error is the same, i.e. s for all the coordinates, then the formula that give us the sE = error of the area simplified and generalized as follows:

$$\mathbf{s}_{\mathrm{E}} = 2.7 \times \mathbf{s}_{\mathrm{P}} \times \sqrt{\mathbf{S}_{\mathrm{m}}} \times \sqrt{\mathbf{U}},\tag{12}$$

where sp (m) is the distance root mean square, Sm (m) is between 10 - 15 m and U (m) is the perimeter of the polygon which is equal to n × Sm, where n is the number of measured points along the boundaries of the polygonal surface area measuring.

We get the following 95% confidence intervals for the three area measurements, with the use of the following formula assuming that the data follow the normal distribution (Gaussian distribution):

$$E - 1,96 S_E \le E \le E + 1,96 S_E,$$
(13)

where  $E(m^2)$  is the area from Gauss formula as derives from the GPS measurements.



These formulas by application of kinematic method in moving vehicle they want a closer inspection and re-confirmation of their reliability. For mixed sets (<30) measurements (deviations) is good for the estimation of confidence limits the t distribution instead of the normal Gauss (Schmidt, 1996).

#### **3 Results - Discussion**

If a GPS receiver displays position coordinates that are different from the "true coordinates" of the antenna position, this is <u>position error</u>. A vast variety of measures have been employed for measuring this error, i.e. the degree of conformance between the estimated or measured position.

Specifically, for each computed position i (i=1,2,...,n), the positional error, say Di, is calculated as the deviation between the satellite obtained coordinate from the GPS receiver and the true reference coordinate.

In the following table 1 descriptive statistics for the overall positional errors of the GPS systems are presented, such as minimum and maximum Di, average Di and standard deviation of Di.

Table 1: Descriptive statistics for the positional errors along the x, y and z axes of all measurements from GPS

Positional Error	Minimum (m)	Maximum (m)	Average Positional Error (m)	Std. Deviation (m)	N
Positional Error for GPS (x axis)	0.011	0.665	0.134	0.120	28
Positional Error for GPS (y axis)	0.015	0.711	0.141	0.171	28
Positional Error for GPS (z axis)	0.001	0.635	0.156	0.165	20
TOTAL	0.001	0.711	0.142	0.151	76

Average positional errors as we observe from the latter results are similar for the three axes, with the x-axis average positional error being the lowest (0.134 m). Total average positional error is 0.142 m.



Tables 2 and 3 present descriptive statistics for the two GPS receivers, separately.

Positional Error	Minimum (m)	Maximum (m)	Average Positional Error (m)	Std. Deviation (m)	N
Positional Error for GPS (x axis)	0.043	0.665	0.212	0.197	8
Positional Error for GPS (y axis)	0.128	0.711	0.348	0.208	8
Positional Error for GPS (z axis)					
TOTAL	0.043	0.711	0.280	0.208	16

Table 2: Descriptive statistics for the positional errors along the x, y and z axes of low-cost GPS

Table 3: Descriptive statistics for the positional errors along the x, y and z axes of DGPS L1

Positional Error	Minimum (m)	Maximum (m)	Average Positional Error (m)	Std. Deviation (m)	N
Positional Error for GPS (x axis)	0.011	0.195	0.103	0.050	20
Positional Error for GPS (y axis)	0.015	0.089	0.058	0.022	20
Positional Error for GPS (z axis)	0.001	0.635	0.156	0.165	20
TOTAL	0.001	0.635	0.106	0.106	60

The following table 4 shows the different accuracy measures calculated.

Table 4: 2D and 3D	positional accurac	y measures for	the data	collected
--------------------	--------------------	----------------	----------	-----------

MEASURES	2D	3D
DRMS	0.209	
2DRMS	0.418	
CEP	0.173	
R95	0.360	
SEP		0.232
MRSE		0.266



As we see, the values of the accuracy measures for GPS are between 0.173 and 0.418. The DRMS is 0.209. By this, we conclude that with the GPS receiver we will fall within 0.209 meters of the true measurement at 65% of the time, indicating thus the precise estimation as concerns the horizontal precision of the GPS.

Accordingly, measurements from GPS with a CEP value of 0.173 will be within 0.173 meters of the true measurement at 50% of the time, while the other 50% of the time the measurements will be in error by more than 0.204 meters.

As concerns the 3-dimensional accuracy (i.e. the combined horizontal and vertical accuracy), the Spherical Error Probable (SEP) is 0.232, whereas the MRSE is 0.266.

The following tables 5 & 6 present the accuracy measures for the two GPS devices separately. Once again, the high accuracy GPS performs better in comparison to the low-cost GPS.

MEASURES	2D	3D
DRMS	0.287	
2DRMS	0.573	
CEP	0.239	
R95	0.498	
SEP		
MRSE		

Table 5: 2D and 3D positional accuracy measures for the low-cost GPS

Table 6: 2D and 3D positional accuracy measures for the DGPS

MEASURES	2D	3D
DRMS	0.055	
2DRMS	0.109	
CEP	0.041	
R95	0.086	
SEP		0.121
MRSE		0.174

In the sequel, data were analyzed in order to validate the effects of various factors on the obtained positional errors, such as the effect of positioning points, type of GPS, and the direction (Northing, Easting and Vertical). In doing this, a Generalized Linear model (GLM) was fitted to the data, where the dependent variable was chosen to be the positional errors, whereas as the independent variables were chosen the above mentioned factors. Table 7 summarizes the obtained results concerning parameter estimates of the fitted model, along with the associated p-values.

#### Table 7: GLM Parameter Significance Tests



# Dependent variable: Positional Error (D<sub>i</sub>)

Parameter	Beta coefficient	p-value	95% confidence interval
intercept	0.161	0.002*	(0.062 - 0.260)
	Type of GPS (n	ref.: DGPS L <sub>1</sub> )	
LOW-COST GPS	0.164	$0.006^*$	(0.047 - 0.280)
	Direction (re	ef.: Vertical)	
Easting	-0.055	0.247	(-0.150 – 0.039)
Northing	-0.063	0.186	(-0.158 – 0.031)
Measurement point	0.000	0.920	(-0.006 - 0.007)
R Square	0.414		
Adjusted R Square	0.374		
N=79			

(\*) Coefficient is significant at a 1% significance level

The above fitted model explained 41.4% of the variation, as indicated by the value of the R2 statistic. As it follows from Table 7, the type of GPS of which measurements taken is a significant factor for the positional error, at a 1% level of significance (beta= 0.164, p-value=0.006<0.01). Indeed, as suggested by the model, the probabilities of lower positional error are increasing by a factor of 0.164 in case of using the DGPS L1, when compared with the low-cost GPS, which gives larger positional errors.

In contrast, positional error among the GPS positioning points did not differed statistically significantly (beta= 0.000, p-value=0.92>0.05) at a 5% level of statistical significance.

Finally, the positional error although that is more apparent in the z-axis positioning, when compared with the x-axis and y-axis, this was not substantiated by the statistical modeling. Nevertheless, both the previous analyses and the magnitude of the model coefficients indicate that measurements taken at vertical positioning provide the largest positional error. The final GLM regression model acquired from the fit is given by the following equation:

 $D_1 = 0.161 + 0.164 \times (LOW \_ COST \_ GPS)_i + e_i$ 

where Di the positional error (m), GPS=1 if GPS is low-cost GPS, and ei stands for the error not explained by the model.

# 3.1 Using GPS for Estimating an Area

The aim of this paper is also to investigate the reliability of using different types of GPS and GIS in order to calculate the area of forest plots. Table 8 presents the estimated measurements taken from the two methods of area measurements in the three areas under investigation, along with the errors derived by the comparison with the true area measurements.

#### Table 8: Errors in area measurements



Total station (m <sup>2</sup> )	Area (m <sup>2</sup> )	GPS &GIS	Error
7,175.988265	7,157.907089	DGPS	18
4,756.8450	4817.09	GPS-GIS	60
391342.82	391298.12	DGPS	45
7,175.988265	7193.85	GIS	18
4,756.845	4748.5	GIS	8
391342.82	393509.97	GIS	2,167
	7,175.988265 4,756.8450 391342.82 7,175.988265 4,756.845 391342.82	Total station (iii )     Artea (iii )       7,175.988265     7,157.907089       4,756.8450     4817.09       391342.82     391298.12       7,175.988265     7193.85       4,756.845     4748.5       391342.82     393509.97	7,175.988265       7,157.907089       DGPS         4,756.8450       4817.09       GPS-GIS         391342.82       391298.12       DGPS         7,175.988265       7193.85       GIS         4,756.845       4748.5       GIS         391342.82       393509.97       GIS

It is evident that the low-cost GPS produced in all three regions the highest errors in measuring correctly the area of the region (average error for the low-cost GPS = 41.34; average error for the DGPS = 9.46).

As a next step, it would be also of interest to test if there is a correlation between the positional errors in measuring coordinates and area measuring. To this end, we have performed the fit of various curves through the use of regression analysis, utilizing as the dependent variable the positional errors for the x-, y- and z-axis, and as independent variable the errors derived by the low-cost and DGPS, respectively for the three regions. The results of the regression analysis showed that there is a statistically significant association between the two error measurements (Table 9).

Model	$\mathbf{R}^2$	F	p-value	intercept	b1	b2	b3
Linear	0.167	14.876	< 0.001	0.093	0.003		
Logarithmic	0.075	6.021	0.016	0.084	0.029		
Inverse	0.029	2.234	0.139	0.175	-0.121		
Quadratic	0.250	12.178	< 0.001	0.137	-0.007	0.000	
Cubic	0.250	12.178	< 0.001	0.133	-0.004	0.000	0.000
Compound	0.098	8.077	0.006	0.067	1.015		
Power	0.020	1.507	0.224	0.071	0.112		
S	0.001	0.043	0.836	-2.394	-0.126		
Growth	0.098	8.077	0.006	-2.707	0.015		
Exponential	0.098	8.077	0.006	0.067	0.015		
Logistic	0.098	8.077	0.006	14.984	0.985		
Dependent Varia	ble: positiona	al error					

Table 9: Regression coefficients for the various curves fitted



The F-test for the significance of each regression model fitted was statistically significant for most of the curves (p-value<0.05) at a 5% level of significance, indicating the significance of the association between the surface error and the positional error. Additionally, R2 statistic shows that the best models to describe this association are the Quadratic, Cubic and Linear, thus there is an evident correlation between the two measurements. It appears that low-cost GPS calculates the areas of forests plots in a similar manner that calculates the coordinates, producing constantly higher errors in comparison to the more accurate GPS devices.

In the sequel, we will try to come up with estimation, from a statistical point of view, of the confidence intervals of the area measurement for each type of GPS. In doing this, first we need to calculate positional errors along the x- and the y-axis, i.e. sx and sy for each area and for each GPS type separately. The calculated errors are shown in the table 10, along with the corresponding measure of Distance Root Mean Square (DRMS) = sp.

Area	Туре	S <sub>x</sub>	Sy	sp
Exohi	GIS	0.197	0.208	0.286
Taxiarchis	GPS	0.057	0.025	0.062
Phinikas	GIS	0.048	0.021	0.052

# Table 10: Distance Root Mean Square, sx and sy

In table 11 we get the following 95% confidence intervals for the three area measurements, and assuming that the data follow the normal distribution (Gaussian distribution).

Area	Туре	Estimated E	95% Lower Limit	95% Upper Limit
Taxiarchis	GIS	7193.85	7172.969	7214.730
Exohi	GPS	4817.09	4726.279	4907.901
Phinikas	GIS	393509.97	393483.219	393536.721

Table 11: 95% CI for the area measurement (Gaussian distribution)

Whereas, by assuming data following the t-distribution of Student (preferred when n<30), and from the tables of t-distribution for the corresponding degrees of freedom, we get the following confidence intervals (Table 12):

Table 12: 95% CI for the area measurement (t-distribution)

Area	Туре	Estimated E	95% Lower Limit	95% Upper Limit	N
Taxiarchis	GIS	7193.85	7175.377	7212.323	16
Exohi	GPS	4817.09	4736.194	4897.986	18
Phinikas	GIS	393509.97	393486.986	393532.954	42



Observe now, that with the use of the t-distribution confidence intervals, we get narrower confidence limits for the area measurement.

# 4 Conclusion

The issue of GPS accuracy can be complex and an ideal description of GPS accuracy will have reference to several factors. In this study we have made an attempt to examine the performance of GPS receivers in the situation of a place on the skirts of the town planning complex As which is forest botanical garden; a rural forest area nearby a suburban forest and last a strictly forest area. The results of the current analysis showed that in general there were no large differences between the measurements of the GPS receiver and the true coordinates regarding accuracy in measuring coordinates. The differences were more apparent in the z- axis and in the y-axis measurement errors, and significantly lower in the x- axis. This was verified using both the average positional error statistic and the hypothesis testing based on the GLM model. The results of the study concerning positional errors found were more or less in accordance with previously conducted analyses.

We have also made an attempt to examine the performance of two different types of GPS receivers, one advanced and highly accurate and one simpler. The results of the analysis showed that there were significant differences between the receivers regarding accuracy and precision in measuring coordinates.

In addition, regression analysis applied to the data to assess which, and how various factors affect the GPS measurement errors. The study demonstrated that the type of GPS receiver had statistically significant association with positional errors in the case of the GPS receiver.

The aim of this paper was also to investigate the reliability of using different types of GPS in order to calculate the area of forest plots. Our analysis showed that low-cost GPS calculates the areas of forests plots in a similar manner that calculates the coordinates, producing constantly higher errors in comparison to the more accurate GPS devices, and we have found that this association is best described by a Quadratic and Cubic curve.

In conclusion, the results of the study concerning positional errors found were more or less in accordance with previously conducted analyses. For instance, as concerns the GPS receiver under study, we have found that the mean positional error of 0.115 meters (SD=0.2 m), whereas Næsset and Jonmeister (2002) report for an analogous low-cost GPS receiver mean positional errors ranging between 0.49 and 3.60 m under forest canopy. Results of similar magnitude were also reported in Yoshimura and Hasegawa (2003). However, there are also studies found in the literature where positional errors using low-cost receivers are substantially higher (see for instance, Rodriguez-Pérez et al., 2006).

# **5** References

American Society of Civil Engineers (Committee on Cartographic Surveying, Surveying and Mapping Division) (1983): Map uses, scales and accuracies for engineering and associated purposes. American Society of Civil Engineers, New York.

American Society of Photogrammetry (Committee for Specifications and Standards, Professional Practice Division) (1985): Accuracy specification for large-scale line maps. *Photogrammetric Engineering and Remote Sensing* 51, 195-199.

Dana, P. H. (1999): Global Positioning System (GPS) time dissemination for Real-Time Applications. *Real-Time Systems* 12, 9-40.



Goodchild, M. (1991): Issues of quality and uncertainty. In Muller, J., Ed., Advances in cartography, *Elsevier Applied Science*, London, 113-139.

Kaplan, E. D. (1996): Understanding GPS principles and applications. Boston, Artech House Inc.

Karsky, D., Chamberlain, K., Mancebo, S., Patterson, D. and Jasumback, T. (2000): Comparison of GPS Receivers under a Forest Canopy with Selective Availability Off. *USDA Forest Service Project Report* 7100.

Næsset, E. (2001): Effects of differential single- and dual-frequency GPS and GLONASS observations on point accuracy under forest canopies. *Photogrammetric Engineering & Remote Sensing* 67, 1021-1026.

Næsset, E. and Jonmeister, T. (2002): Assessing point accuracy of DGPS under forest canopy before data aquisition, in the field and after post processing. *Scandinavian Journal of Forest Research* 17, 351-358.

Resnik, B. 2002: Praktische Untersuchungen zur Genauigkeit von Flächenermittlungen mit Handheld-GPS-Empfängern. AVN - Allgemeine Vermessungs-Nachrichten, 10, 346-351.

Rodriguez-Pérez, J.R., Alvarez, M.F., Sanz E. and Gavela, A. (2006): Comparison of GPS Receiver Accuracy and Precision in Forest Environments. Practical Recommendations Regarding Methods and Receiver Selection. *GNSS Processing and Applications*.

Sawaguchi, I., Nishida, K., Shishiuchi, M., and Tatsukawa, S. (2003): Positioning precision and sampling number of DGPS under forest canopies. *Journal of Forest Research* 8, 133–137.

Schmidt, H. 1996: Zur Problematik von Korrelationsschätzungen und Genauigkeitsangaben in der Meßtechnik. *Allgemeine Vermessungs-Nachrichten*, AVN 103, 225 - 232.

Shearer, J. (1990): Accuracy of digital terrain models, In Petrie, G, Kennie, T., Eds., Terrain Modeling in Surveying and Civil Engineering, Thomas Telford, London, 315-336.

Tachiki, Y., Yoshimura, T., Hasegawa, H., Mita, T., Sakai, T and Nakamura, G. (2005): Effects of Polyline Simplification of Dynamic GPS Data under Forest Canopy on Area and Perimeter Estimations. *J For Res* 10, 419-427.

Yoshimura, T. and Hasegawa, H. (2003): Comparing the precision and accuracy of GPS positioning in forested areas. *Journal of Forest Research* 8, 147–152.