

# A Bayesian Analysis of Lactation Curves via Hierarchical Non-Linear Models

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## Introduction

- In the present study a Bayesian analysis on lactation curve is performed. Analysis is performed on daily milk yield base. The data are collected from an intensive-type sheep farm located in the region of Volos, Greece. The farm breeds the local Greek sheep breed of Chios.
- Shape of the curve provides important information on biological and economic capacity of the animal or herd under study.
- Why lactation curve?
  - Test hypothesis on the behavior of mammary gland machinery assisting the research from physiologist, nutritionists and geneticists.
  - Assist in management, for instance in taking decisions on issues such as milking, nutrition and veterinarian treatment strategies.
  - Can be used to predict future milk yields of an individual animal or a herd (Sherchand *et al.*, 1995).

## Objectives

- We investigate the application of Bayesian analysis to test day records of milk, based on the assumption of Wood's lactation curve.
- The proposed function is of the following form:

$$f(t) = at^b \exp(-ct)$$

- Parameter  $\alpha$  approximates the level at which the milk production begins at birth.  $b$  represents the rate of increase up to maximum performance.  $c$  represents the rate of decline after reaching the maximum yield of milk (Wood, 1972).

- Wood's model was an obvious choice because it has been used in most modeling studies of dairy farms in both cows and sheep. In addition, and for the specific data we performed a preliminary analysis (Karangeli, 2012) using other popular mathematical models for describing lactation curves it appears that the Wood curve seemed like the preferred choice for describing the data at hand.

## Materials and Methods

- We provide inference by fitting different structures and performing model selection through the Deviance information criterion (DIC).
- Correlations among parameters of Wood's model are assumed for the fit of a specific structure to assess the effects of genetic selection on the shape of lactation curve.
- Data used were collected from the "AMNOS SA" farm located in the industrial area of Volos, Magnesia. The milk yield data originate from a milking parlour with an automatic identification and milk recording system. No weaning is performed in the farm so the whole lactation curve is collected.

- Wood's model assumes that test-day record  $y_{ij}$  of ewe  $i$  at time  $j$  ( $i=1,2,\dots,n$ ;  $j=1,2,\dots,m$ ) can be represented as:

$$y_{ij} = f(\theta_i, t_{ij}) + e_{ij}$$

where  $\theta_i = (\alpha_i, b_i, c_i)'$

- Restrictions on the parameters  $\alpha$ ,  $b$  and  $c$  are:  $\alpha > 0$ ,  $0 < b < 1$  and  $0 < c < 1$ .

- Residuals  $e_{ij}$  are assumed to be independent and Normally distributed, with their variance following an inverse Gamma distribution, with  $1/\sigma_e^2 \sim \text{Gamma}(0.001, 0.001)$

- We fit to the daily lactation yield data a fixed effects Wood curve, an independent curve model, a random-effects model and a multivariate random-effects model that additionally to the previous three models assuming independence of the model's parameters, now we assume that  $\alpha$ ,  $b$  and  $c$  are correlated to each other.

- Prior distributions:  $b \sim U(0,1)$ ;  $c \sim U(0,1)$ ;  $\ln(\alpha) \sim N(0, K)$ ,  $K \rightarrow \infty$ .

- Covariate information is assumed to affect the milk yield (effects of season of calving, and year effects). We have used WinBUGS (Spiegelhalter *et al.*, 2002) to implement the Bayesian approach. Burn-in period was 5,000 iterations. Monitor a large chain of 100,000 iterations (one in 10 samples kept), so the marginal density estimations from each one of the parameters to be based on a sample of 10,000.

## Results and discussion

**Table 1:** Mean Parameters of the lactation curve

MODEL	$\alpha$	$b$	$c$	$\sigma_e$
Independent curves	791.35 (523.61-1247.01)	0.49 (0.312-0.65)	0.013 (0.014-0.015)	356 (353.2-358.8)
Fixed effects	1025 (991.7-1057)	0.305 (0.295-0.316)	0.009 (0.009-0.0094)	561.4 (556.9-565.8)
Random effects	808.84 (560.49-1154.17)	0.436 (0.314-0.568)	0.012 (0.009-0.014)	357.9 (355.2-360.8)
Multivariate Random Effects	804.25 (534.49,1567.54)	0.467 (0.22,0.622)	0.012 (0.009,0.016)	356.7 (353.8,364.8)

Marginal posterior distributions for  $\alpha$ ,  $b$  and  $c$  are summarized in Table 1.

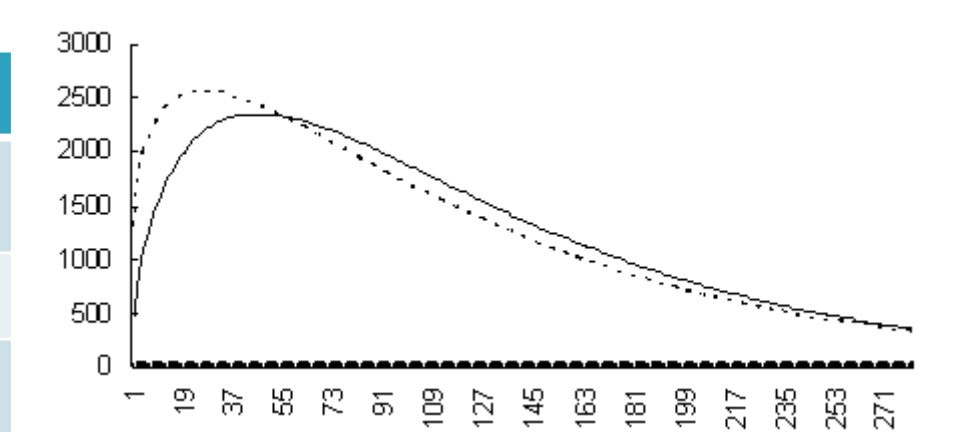
**Table 2:** Correlations between  $\alpha$ ,  $b$  and  $c$  from multivariate model

	$\alpha$	$b$	$c$
$\alpha$	1		
$b$	-0.929 (-0.946,-0.868)	1	
$c$	-0.467 (-0.567,-0.342)	0.657 (0.564,0.73)	1

- Correlations between the parameters were statistically important.
- Correlations of  $\alpha$  with  $b$  and  $c$  were negative.
- $b$  and  $c$  were positively correlated.
- Biological interpretation of these results is also conceptually valid.

**Table 3:** Fit statistics

Model	Deviance	pD	DIC
Independent curves	463,861	934.79	464,796
Fixed effects	492,822	3.932	492,826
Random effects	464,207	751.328	464,958
Multivariate Random Effects	466,294	-18,279.6	448,014



**Figure 1.** A typical lactation curve according to Wood's model

- Next, we consider covariate information, comprised of year/season effects.
- We were interested in if and how season/year affects  $\alpha$ ,  $b$  and  $c$  parameters.
- $\alpha$ ,  $b$  and  $c$  parameters were all affected by year/season effects.
- Most significant effects are those of spring 2009, winter 2010 and autumn 2009 seasons, as indicated by at least three out of the four models tested.
- During these seasons initial yield ( $\alpha$ ) and ascent to peak ( $b$ ) increase, whereas descent to peak ( $c$ ) decreases.

- Effects of year/season were more divergent on parameter  $b$ .

**Table 4:** Fit statistics (including covariate information)

Model	Deviance	pD	DIC
Independent curves	463,868	-9,102.46	454,766
Fixed effects	491,894	-3.773	491,890
Random effects	464,211	730.582	464,941
Multivariate Random Effects	463,961	-680.375	463,281

- Random effects /fixed effects models remain almost unaffected by the inclusion of animal random effects and year/season effect.

- Independent curves model significantly improves its performance (DIC=454,766).

- Multivariate random effects model increases its DIC value (DIC=463,281)

## Conclusions

- The multivariate random effects model taking into account possible correlations between the three parameters of the Wood function, was the best model when no covariate information was introduced into the models.

- Independent curves model was the best model with the inclusion of covariate information, with multivariate being the second best model.

- Fixed effects model showed the worst fit to the data.

- The procedures implemented here provided estimates for individual parameters of the lactation curves, using all available information from the data. With this approach, the number of non-estimated lactation curves is dramatically reduced, compared to alternative analyses.

## References

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