Stochastic spatio-temporal modeling with applications to animal infectious diseases

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Motivating example

- 2 cases of epidemics in livestock from the Evros prefecture

- A) foot-and-mouth disease (FMD)
- B) sheep pox disease

Characteristics of epidemic data

- Spatio-temporal dependence
- Environmental noise
- Multicollinearity
- Presence of “excess” zeros

Foot-and-mouth disease

- A viral disease, infecting mainly cattle, sheep, goats, pigs.
- Infection results in:
  - Reduced productivity (up to 70%)
  - Death (rare, mainly for young animals)

Transmission:

• Direct contact (animal-to-animal)

•Indirect contact (people, vehicles, etc.)

•Airborne disease (less effective)

Epidemics in Evros, Greece

2 major epidemics in Evros region

- FMD epidemic (during July-September, 2000)
- Sheep pox endemic (1994-1998)

- FMD: ~10.000 dead livestock
- Sheep pox : ~35.500 dead livestock

Models for FMD & sheep pox

- Data:
  - y_t: total cases of disease occurrence for sheep pox/FMD (case: each infected farm)
  - t: week/day for sheep pox/FMD
  - Spatial information in the form of coordinates (x_i, y_i) for each farm i.

Explanatory Variables

X’s:

Covariates referring to environmental/meteorological data

- Temperature levels (min, mean, max)
- Rainfall
- Humidity
- Soil temperature (10cm)
- Wind speed

Other predictors:

- Spatial kernels
- Parameter τ: $y_t = \tau * y_{t-1}$

Basic features of our modeling

- Point processes accounting for “excess” zeros
- Regression based upon a series of (environmental) covariates
- Stochastic component: Ornstein-Uhlenbeck (OU) process
- Spatial distance kernels
- Bayesian g-priors for dealing with correlated covariates
- A link to epidemic control

Model formulation

$$\begin{cases} y_i & \sim g(y_i|\theta_i, p_i) \\ g(y_i|\theta_i, p_i) & = p_i I_{\{y_i=0\}} + (1-p_i) f(y_i|\theta_i) \\ \theta_i & = h(\lambda_i) = \exp(\lambda_i) \\ d\lambda_t & = \phi(\lambda_t - \mu_t)dt + dB_t \end{cases}$$

Where B_t denotes Brownian motion, and μ_t is given by:

$$\mu_t = \mathbf{X}_{(i)}\boldsymbol{\beta} + \tau \cdot y_{i-1} + K\left(d_i, \boldsymbol{\Theta}_K\right)$$

$$\log\left(\frac{p_t}{1-p_t}\right) = \mathbf{X}^t_{(i)} \cdot \boldsymbol{\beta}^z + \tau^z \cdot y_{i-1} + K\left(d_i, \boldsymbol{\Theta}^z_{\mathbf{K}}\right)$$

p_i ($0 \le p_i \le 1$) is the percentage of excess zeros at time t_i.

Model framework

- Poisson, negative binomial, ZIP, ZINB are special cases of the above formulation.

The O-U process

- λ_t: an Ornstein-Uhlenbeck process around μ_t which in turn is determined by the covariates and kernels.

$$\lambda_{t_{i+1}} | \lambda_{t_i} \sim N\left(\mu^{(i)} + \left(\lambda_{t_i} - \mu^{(i)}\right) e^{-\phi\delta_i},\ \frac{1 - e^{-2\phi\delta_i}}{2\phi}\right),\quad \delta_i = t_{i+1} - t_i.$$

With each change in the covariates we have a shock to the system, of which the process λ_t adapts through the OU process, with rate of convergence driven by φ (Struthers and McLeish, 2011).

Spatial information

$$K\left(d_i,\boldsymbol{\Theta}_{\mathbf{K}}\right)=\begin{cases}\dfrac{1}{\left|d_i\right|}\displaystyle\sum_{k\in S_i}\sum_{k\in S_{i-1}}\mathrm{K}\left(d_{k\ell},\boldsymbol{\Theta}_{\mathbf{K}}\right) & \text{if } y_i>0 \text{ and } y_{i-1}>0\\ \mathrm{K}\left(1,\boldsymbol{\Theta}_{\mathbf{K}}\right) & \text{if } y_i>0 \text{ and } y_{i-1}=0\\ \mathrm{K}\left(d_{\min},\boldsymbol{\Theta}_{\mathbf{K}}\right) & \text{if } y_i=0\end{cases}$$

K(•): predetermined function of average distance between farms of previous and current week/day (kernel functions).

d_{min}: minimum distance beyond which there is no transmission of disease

Summary of kernel functions compared

Table 1

Summary of transmission kernel functions included in spatio-temporal models.

Notation	$\mathcal{K}(\mathsf{d}_{k\ell}, \Theta_K)$	Θ_K	Reference
A	$\left(1 + \frac{\mathsf{d}_{k\ell}}{a}\right)^{-c}$	(a, c)	Chis-Ster and Ferguson (2007)
B	$\exp\left\{-\left(\frac{\mathsf{d}_{k\ell}}{a}\right)^{c}\right\}$	(a, c)	Keeling (2001)
C	$\exp\left\{-\left(\frac{\mathsf{d}_{k\ell}}{a}\right)^{c}\right\} + r$	(a, c, r)	Diggle (2006)
D	$a \exp\left(-a\, \mathsf{d}_{k\ell}\right)$	a	Szmaragd (2009)
E	$\frac{\alpha}{\sqrt{\pi}} \exp\left(-a^2 \mathsf{d}_{k\ell}^2\right)$	a	Szmaragd (2009)
F	$\frac{a}{4} \exp\left(-a^{\frac{1}{2}} \mathsf{d}_{k\ell}^{\frac{1}{2}}\right)$	a	Szmaragd (2009)

Variable selection

- We utilize the hyper g-prior (Liang, 2008), modified by Bové and Held (2011) for GLMs.

- Following Ntzoufras et al. (2003) use a slightly modified version of hyper g-prior that assigns a Beta density to the shrinkage factor g/(1+g) as:

$$\frac{g}{1+g} \sim \ Beta\left(1, \frac{\alpha}{2} - 1\right).$$

Variable selection

- We focus on hyper g-prior with α=4 however we employ sensitivity analysis for various α (α϶[2, 4]).
- We also compare with other g priors (sensitivity analysis):
  - Hyper g/n prior
  - Zellner's g prior (g=n)
  - Zellner's g prior (g=p^2)
  - Empirical normal prior

Decomposition of infection rate

- Lending ideas from Meyer et al. (2012), we split the infection rate λ_t to endemic/epidemic components:

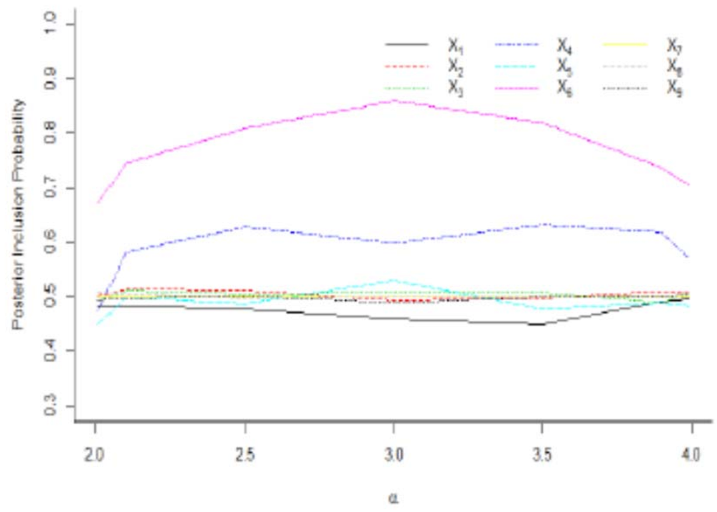
$$\lambda = \lambda_{endemic}\, \lambda_{epidemic} = \exp(\theta_{endemic} + \theta_{epidemic})$$

- Endemic→ meteorological covariates
- Epidemic→ spatial kernels, # of cattle, sheep

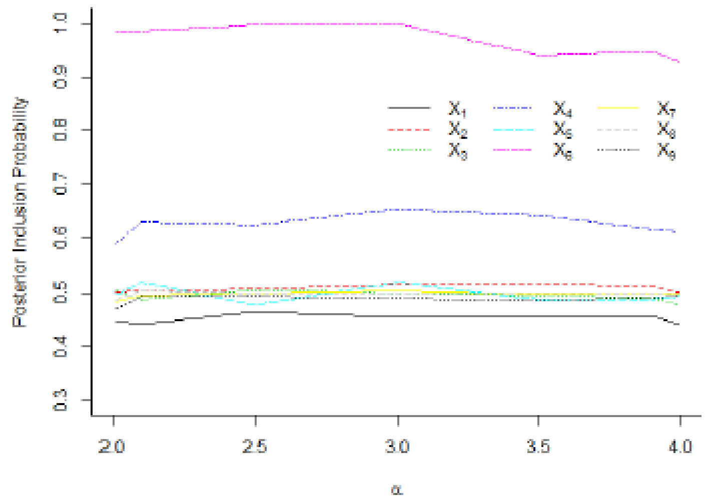
Epidemic control

- Connection of the stochastic model with a suitable Branching Process
- Estimate probability of extinction (q)
- For Poisson distribution, the q's are calculated by solving: $\exp(q\lambda)=q\exp(\lambda)$
- Extend the above for the ZIP model by: $q=\min\{1,q(\lambda)+p\}$

Results – covariate selection (sheep pox)

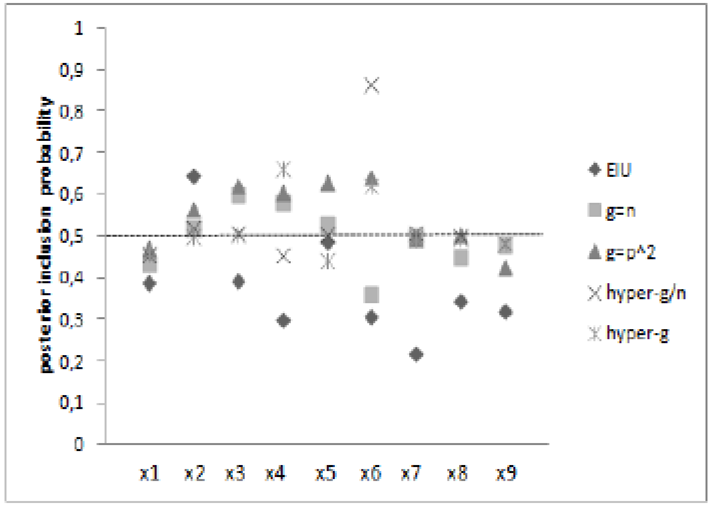


Sensitivity analysis for various α (hyper g-prior) for λ_t.



Sensitivity analysis for various α (hyper g/n-prior) for λ_t.

Results – covariate selection (sheep pox)



Graph presents posterior inclusion probabilities for the covariates under the various g-prior approaches (infection rate λ_t).

The results refer to applying a uniform prior for inclusion probabilities γ:

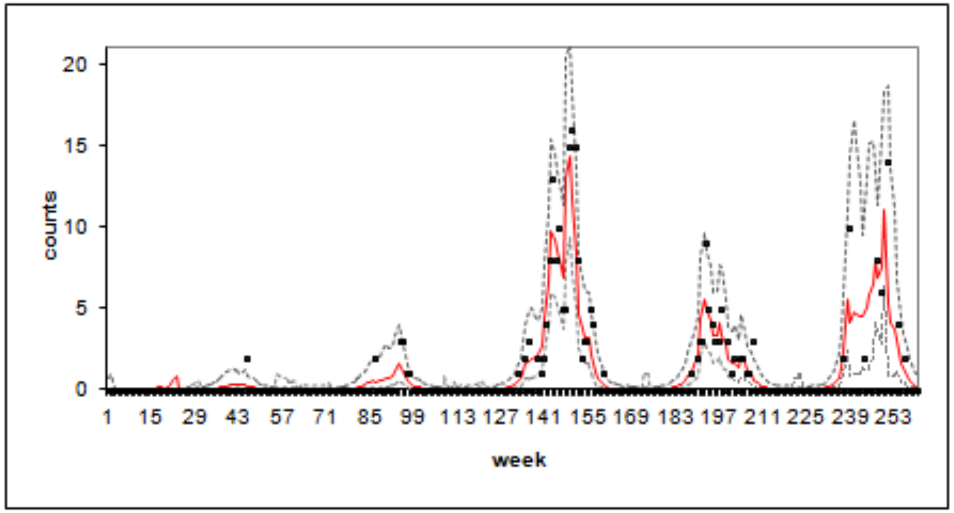
$$\gamma_j \sim \text{Bernoulli}(0.5).$$

Η εφαρμογή μιας beta-binomial prior $\gamma_j \sim \text{Bernoulli}(p) \quad p \sim \text{Beta}(1,1)$; έδωσε παρόμοια κατάταξη των covariates, αυξάνοντας όμως τις εκ των υστέρων πιθανότητες επιλογής για όλες τις μεταβλητές.

Results – model selection (sheep pox)

Model	$\bar{D}$
Poisson	310.2
Negative binomial	367.5
ZIP	277.3
ZIP_h	**270**
ZINB	401.6

Kernel	$\bar{D}$
A Chis-Ster and Ferguson (2007)	**228**
B Keeling et al. (2001)	228.5
C Diggle (2006)	231.3
D Szmaragd et al. (2009)	240.4
E Szmaragd et al. (2009)	231.9
F Szmaragd et al. (2009)	234.6

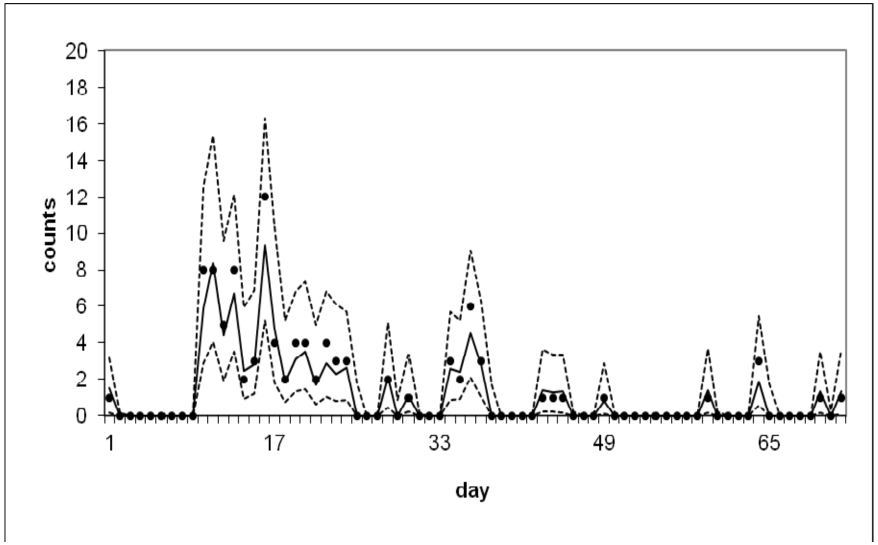


Adding spatial information (under ZIP distribution) and utilizing the proposed OU formulation we achieve important improvement in model fit.

Results – model selection (foot-and-mouth)

Model	$\bar{D}$
Poisson	145.5
Negative binomial	149
ZIP	141.3
ZIP_h	**124.5**
ZINB	145.2

Kernel	$\bar{D}$
A Chis-Ster and Ferguson (2007)	115.2
B Keeling et al. (2001)	115.6
C Diggle (2006)	114.2
D Szmaragd et al. (2009)	113.5
E Szmaragd et al. (2009)	**113.4**
F Szmaragd et al. (2009)	116.8



Similar improvement in model fit (ZIP distribution).

Results – model selection (sheep pox)

Model	Computational time		% of reduction in computational time
	Taylor model (Choi et al., 2012)	SM model	
Poisson	5.456 s	938 s	82.8%
Negative binomial	3.489 s	986 s	71.7%
ZIP	3.539 s	917 s	74.09%
ZIPh	3.688 s	913 s	75.25%
ZINB	3.956 s	967 s	75.56%

- The OU component, under the proposed formulation reduces significantly the computational time for all models, in addition to having a much better fit.

- Reductions of similar magnitude in the running times were also observed for the FMD data.

Results

Sheep pox data

- Infection rate (λ_t) is affected by:
  - Max temperature (+)
  - Humidity (-)
  - Distance
- "Excess" zero probability (p_t) is affected by:
  - Min temperature (+)
  - Humidity (+)
  - Distance

Foot-and-mouth data

- Infection rate (λ_t) is affected by:
  - Distance
- "Excess" zero probability (p_t) is affected by:
  - # of cattle (-)
  - # of sheep (-)
  - Distance

Results

- No effects for the meteorological covariates on FMD occurrences

- Conversely, temperature and humidity are significant for sheep pox occurrence

- *Sheep pox*: the movement from low to high temperatures probably increases incidence of sheep pox.
- Humidity reduces incidence of sheep pox

Results

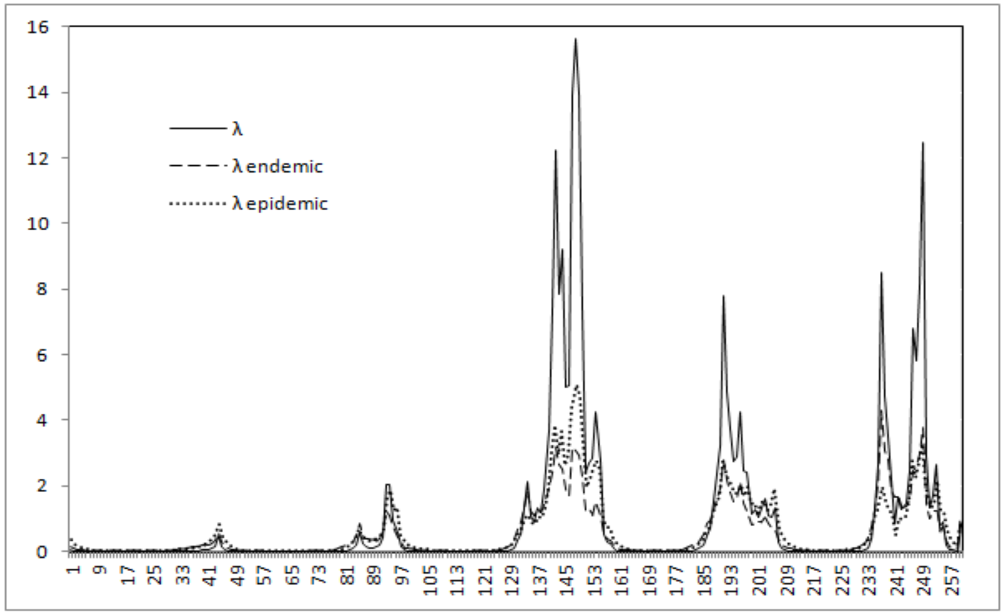
Possible reason for this:

- FMD → epidemic outbreak (duration: July-September, 2000)
- Sheep pox → endemic (duration: 1994-98)

Results – decomposition of λ_t (sheep pox)

	min	median	max
$\Theta_{endemic}$	1.961 (1.904-2.017)	4.045 (4.021-4.068)	16.506 (16.309-16.703)
$\Theta_{epidemic}$	1.269 (1.256-1.282)	1.156 (1.145-1.166)	1.011 (1.01-1.013)

Scenarios for future epidemic

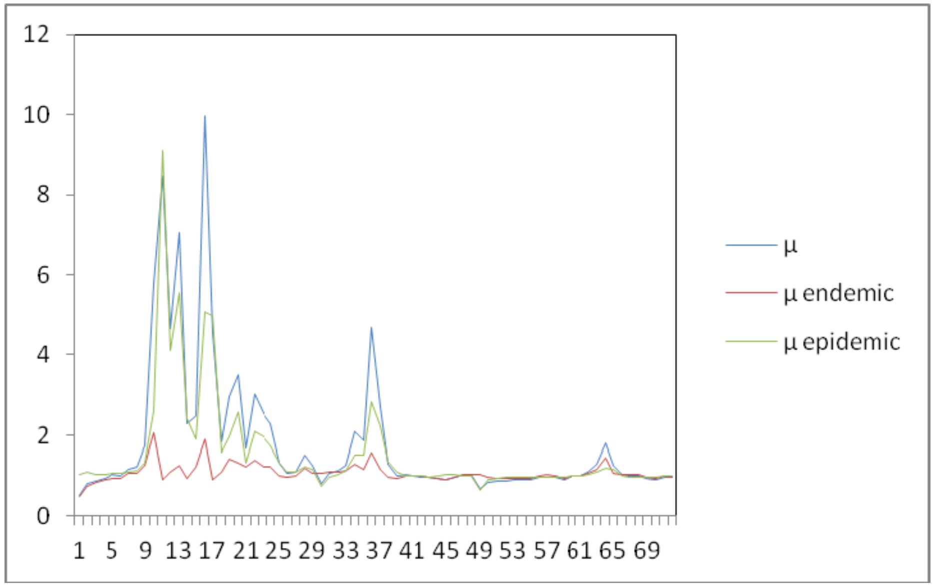


Epidemic of 1994-1998

Results – decomposition of λ_t (foot-and-mouth)

	min	median	max
$\Theta_{endemic}$	0.987 (0.983; 0.955)	0.979 (0.977; 0.98)	0.19 (0.069; 0.311)
$\Theta_{epidemic}$	0.184 (0.165; 0.203)	0.572 (0.548; 0.595)	0.831 (0.808; 0.854)

Scenarios for future epidemic



Epidemic outbreak of 2000

Results – epidemic control

q's and 95% credible intervals for scenarios of a future epidemic
Hypothetical scenarios for future outbreaks (early stages of epidemic)

FMD data

	min temperature	average temperature	max temperature	distance
min	0.567 (0.542; 0.592)	0.98 (0.972; 0.987)	0.536 (0.511; 0.562)	0.152 (0.135; 0.168)
max	0.56 (0.536; 0.585)	0.023 (0.017; 0.31)	0.577 (0.553; 0.601)	0.572 (0.548; 0.595)
All covariates at median values	0.564 (0.54; 0.587)			

Sheep pox data

	humidity	maximum temperature	distance
min	0.001 (0.0007-0.001)	0.878 (0.861-0.895)	0.018 (0.01-0.256)
max	0.485 (0.456-0.513)	0.076 (0.062-0.091)	0.195 (0.174-0.217)
all covariates at median values	0.173 (0.153-0.194)		

Discussion

- Development of appropriate stochastic model, deals with “excess” zeros
- Link with policy-focused models
- Largely insensitive to the specific choice of kernel function
- Intuitive decomposition to endemic and epidemic components.

Limitations – future work

- Used deviance-based measures for model selection
- More natural/intuitive approach due to the sequential nature of the data: prequential methodology
- Also compare with recently developed information criteria due to Watanabe

Some references

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THANK YOU FOR YOUR ATTENTION