Ethnomathematics and its Place in the History and Pedagogy of Mathematics

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I. Introductory remarks

In this paper we will discuss some basic issues which may lay the ground for an historical approach to the teaching of mathematics in a novel way. Our project relies primarily on developing the concept of *ethnomathematics.*

Our subject lies on the borderline between the history of mathematics and cultural anthropology. We may conceptualize ethnoscience as the study of scientific and, by extension, technological phenomena in direct relation to their social, economic and cultural backgrounds [1]. There has been much research already on ethnoastronomy, ethnobotany, ethnochemistry, and so on. Not much has been done in ethnomathematics, perhaps because people believe in the universality of mathematics. This seems to be harder to sustain as recent research, mainly carried on by anthropologists, shows evidence of practices which are typically mathematical, such as counting, ordering, sorting, measuring and weighing, done in radically different ways than those which are commonly taught in the school system. This has encouraged a few studies on the evolution of the concepts of mathematics in a cultural and anthropological framework. But we consider this direction to have been pursued only to a very limited and — we might say — timid extent. A basic book by R.L. Wilder which takes this approach and a recent comment on Wilder's approach by C. Smorinski [2] seem to be the most important attempts by mathematicians. On the other hand, there is a reasonable amount of literature on this by anthropologists. Making a bridge between anthropologists and historians of culture and mathematicians is an important step towards recognizing that different modes of thoughts may lead to different forms of mathematics; this is the field which we may call ethnomathematics.

Anton Dimitriu's extensive history of logic [3] briefly describes Indian and Chinese logics merely as background for his general historical study of the logics that originated from Greek thought. We know from other sources that, for example, the concept of "the number one" is a quite different concept in the Nyaya-Vaiśesika epistemology: "the number one is eternal in eternal substances, whereas two, etc., are always non-eternal," and from this proceeds an arithmetic [4, p. 119]. Practically nothing is known about the logic underlying the Inca treatment of numbers, though what is known through the study of the "quipus" suggests that they used a mixed qualitative-quantitative language [5].

These remarks invite us to look at the history of mathematics in a broader context so as to incorporate in it other possible forms of mathematics. But we will go further than these considerations in saying that this is not a mere academic exercise, since its implications for the pedagogy of mathematics are clear. We refer to recent advances in theories of cognition which show how strongly culture and cognition are related. Although for a long time there have been indications of a close connection between cognitive mechanisms and cultural environment, a reductionist tendency, which goes back to Descartes and has to a certain extent grown in parallel with the development of mathematics, tended to dominate education until recently, implying a culture-free cognition. Recently a holistic recognition of the interpenetration of biology and culture has opened up a fertile ground of research on culture and mathematical cognition (see, for example, [6]). This has clear implications for mathematics education, as has been amply discussed in [7] and [8].

II. An historical overview of mathematics education

Let us look very briefly into some aspects of mathematics education throughout history. We need some sort of periodization for this overview which corresponds, to a certain extent, to major turns in the socio-cultural composition of Western history. (We disregard for this purpose other cultures and civilizations.)

Up to the time of Plato, our reference is the beginning and growth of mathematics in two clearly distinct branches: what we might call "scholarly" mathematics, which was incorporated in the ideal education of Greeks, and another, which we may call "practical" mathematics, reserved to manual workers mainly. In the Egyptian origins of mathematical practice there was the space reserved for "practical" mathematics behind it, which was taught to workers. This distinction was carried on into Greek times and Plato clearly says that "all these studies [ciphering and arithmetic, mensurations, relations of planetary orbits] into their minute details is not for the masses but for a selected few" [9, *Laws* VII, 818] and "we should induce those who are to share the highest functions of State to enter upon that study of calculation and take hold of it,... not for the purpose of buying and selling, as if they were preparing to be merchants or hucksters" [9, *Republic* VII 525b]. This distinction between scholarly and practical mathematics, reserved for different social classes, is carried on by the Romans with the "trivium" and "quadrivium" and a practical training for laborers. In the Middle Ages we

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begin to see a convergence of both in one direction: that is, practical mathematics begins to use some ideas from scholarly mathematics in the field of geometry. Practical geometry is a subject in its own right in the Middle Ages. This approximation of practical to theoretical geometry follows the translation from the Arabic of Euclid's *Elements* by Adelard of Bath, (early 12th century). Dominicus Gomdissalinus, in his classification of sciences, says that "it would be disgraceful for someone to exercise any art and not know what it is, and what subject matter it has, and the other things that are premised of it," as cited in [10, p. 8]. With respect to ciphering and counting, changes start to take place with the introduction of Arabic numerals; the treatise of Fibonnaci [11, p. 481] is probably the first to begin this mixing of the practical and theoretical aspects of arithmetic.

The next step in our periodization is the Renaissance when a new labor structure emerges: changes take place in the domain of architecture since drawing makes plans accessible to bricklayers, and machinery can be drawn and reproduced by others than the inventors. In painting, schools are found to be more efficient and treatises become available. The approximation is felt by scholars who start to use the vernacular for their scholarly works, sometimes writing in a non-technical language and in a style accessible to non-scholars. The best known examples may be Galileo, and Newton, with his "Optiks".

The approximation of practical mathematics to scholarly mathematics increases in pace in the industrial era, not only for reasons of necessity in dealing with increasingly complex machinery and instruction manuals, but also for social reasons. Exclusively scholarly training would not suffice for the children of an aristocracy which had to be prepared to keep its social and economical predominance in a new order [11, p. 482]. The approximation of scholarly mathematics and practical mathematics begins to enter the school system, if we may so call education in these ages.

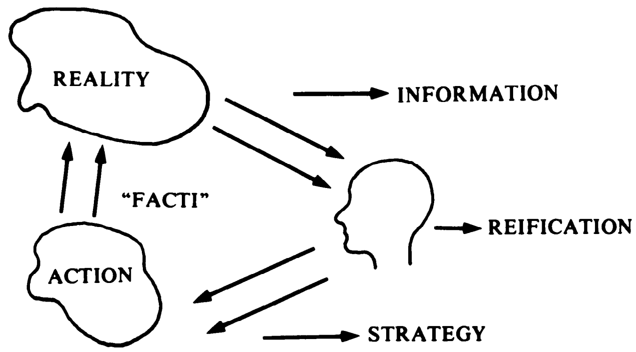
Finally, we reach a last step in this rough periodization in attaining the 20th Century and the widespread concept of mass education. More urgently than for Plato the question of *what* mathematics should be taught in mass educational systems is posed. The answer has been that it should be a mathematics that maintains the economic and social structure, reminiscent of that given to the aristocracy when a good training in mathematics was essential for preparing the elite (as advocated by Plato), and at the same time allows this elite to assume effective management of the productive sector. Mathematics is adapted and given a place as "scholarly practial" mathematics which we will call, from now on, "academic mathematics", i.e., the mathematics which is taught and learned in the schools. In contrast to this we will call *ethnomathematics* the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on. Its identity depends largely on focuses of interest, on motivation, and on certain codes and jargons which do not belong to the realm of academic mathematics. We may go even further in this concept of ethnomathematics to include much of the mathematics which is currently practised by engineers, mainly calculus, which does not respond to the concept of rigor and formalism developed in academic courses of calculus. As an example, the Sylvanus Thompson approach to calculus may fit better into this category of ethnomathematics. And builders and well-diggers and shack-raisers in the slums also use examples of ethnomathematics.

Of course this concept asks for a broader interpretation of what mathematics is. Now we include as mathematics, apart from the Platonic ciphering and arithmetic, mensuration and relations of planetary orbits, the capabilities of classifying, ordering, inferring and modelling. This is a very broad range of human acitivities which, throughout history, have been expropriated by the scholarly establishment, formalized and codified and incorporated into what we call academic mathematics. But which remain alive in culturally identified groups and constitute routines in their practices.

III. Ethnomathematics in history and pedagogy and the relations between them

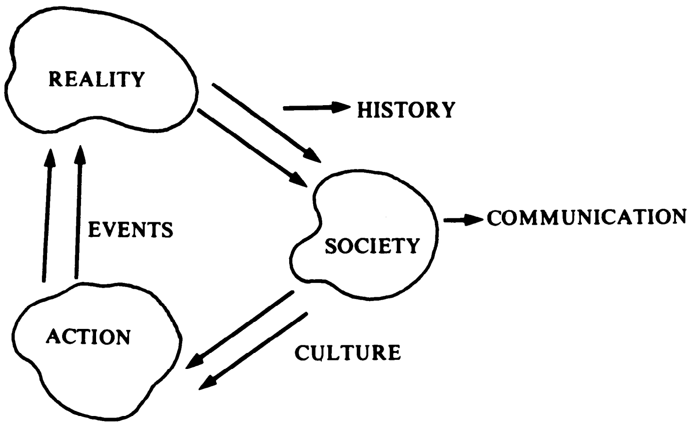
We would like to insist on the broad conceptualization of mathematics which allows us to identify several practices which are essentially mathematical in their nature. And we also presuppose a broad concept of *ethno-*, to include all culturally identifiable groups with their jargons, codes, symbols, myths, and even specific ways of reasoning and inferring. Of course, this comes from a concept of culture as the result of an hierarchization of behavior, from individual behavior through social behavior to cultural behavior.

The concept relies on a model of individual behavior based on the cycle... reality → individual → action → reality..., schematically shown as



In this holistic model we will not enter into a discussion of what is realtiy, or what is an individual, or what is action. We refer to [12]. We simply assume reality in a broad sense, both natural, material, social and psycho-emotional. Now, we observe that links are possible through the mechanism of information (which includes both sensorial and memory, genetic and acquired systems) which produces stimuli in the individual. Through a mechanism of reification these stimuli give rise to strategies (based on codes and models) which allow for action. Action impacts upon reality by introducing facti into this reality, both artifacts and "mentifacts". (We have introduced this neologism to mean all the results of intellectual action which do not materialize, such as ideas, concepts, theories, reflections and thoughts.) These are added to reality, in the broad sense mentioned above, and clearly modify it. The concept of reification has been used by sociobiologists as "the mental activity in which hazily perceived and relatively intangible phenomena, such as complex arrays of objects or activities, are given a factitiously concrete form, simplified and labelled with words or other symbols" [13, p. 380]. We assume this to be the basic mechanism through which strategies for action are defined. This action, be it through artifacts or through mentifacts, modifies reality, which in turn produces additional information which, through this reificative process, modifies or generates new strategies for action, and so on. This ceaseless cycle is the basis for the theoretical framework upon which we base our ethnomathematics concept.

Individual behavior is homogenized in certain ways through mechanisms such as education to build up societal behavior, which in turn generates what we call *culture*. Again a scheme such as



allows for the concept of culture as a strategy for societal action. Now, the mechanism of reification, which is characteristic of individual behavior, is replaced by communication, while information, which impacts upon an individual, is replaced by history, which has its effect on society as a whole. (We will not go deeper here into this theoretical framework; this will appear somewhere else.)

As we have mentioned above, culture manifests itself through jargons, codes, myths, symbols, utopias, and ways of reasoning and inferring. Associated with these we have practises such as ciphering and counting, measuring, classifying, ordering, inferring, modelling, and so on, which constitute ethnomathematics.

The basic question we are then posed is the following: How "theoretical" can ethnomathematics be? It has long been recognized that mathematical practices, such as those mentioned in the end of the previous paragraph, are known to several cuturally differentiated groups; and when we say "known" we mean in a way which is substantially different from the Western or academic way of knowing them. This is often seen in the research of anthropologists and, even before ethnography became recognized as a science, in the reports of travellers all over the world. Interest in these accounts has been mainly curiosity or the source of anthropological concern about learning how natives think. We go a step further in trying to find an underlying structure of inquiry in these *ad hoc* practices. In other terms, we have to pose the following questions:

1. How are *ad hoc* practices and solution of problems developed into methods?
2. How are methods developed into theories?
3. How are theories developed into scientific invention?

It seems, from a study of the history of science, that these are the steps in the building-up of scientific theories. In particular, the history of mathematics gives quite good illustrations of steps 1, 2 and 3, and research programs in the history of science are in essence based on these three questions.

The main issue is then a methodological one, and it lies in the concept of history itself, in particular of the history of science. We have to agree with the initial sentence in Bellone's excellent book on the second scientific revolution: "There is a temptation hidden in the pages of the history of science — the temptation to derive the birth and death of theories, the formalization and growth of concepts, from a scheme (either logical or philosophical) always valid and everywhere applicable... Instead of dealing with real problems, history would then become a learned review of edifying tales for the benefit of one philosophical school or another" [14, p. 1]. This tendency permeates the analysis of popular practices such as ethnoscience, and in particular ethnomathematics, depriving it of any history. As a consequence, it deprives it of the status of knowledge.

It is appropriate at this moment to make a few remarks about the nature of science nowadays, regarded as a large scale professional activity. As we have already mentioned, it developed into this position only since early 19th century. Although scientists communicated among themselves, and scientific periodicals, meetings and associations were known, the activity of scientists in earlier centuries did not receive any reward as such. What reward there was came more as the result of patronage. Universities were little concerned with preparing scientists or training individuals for scientific work. Only in the 19th century did becoming a scientist start to be regarded as a professional activity. And out of this change, the differentiation of science into scientific fields became almost unavoidable. The training of a scientist, now a professional with specific qualifications, was done in his subject, in universities or similar institutions, and mechanisms to qualify him for professional activity were developed. And standards of evaluation of his credentials were developed. Knowledge, particularly scientific knowledge, was granted a status which allowed it to bestow upon individuals the required credentials for their professional activity. This same knowledge, practiced in many strata of society at different levels of sophistication and depth, was expropriated by those who had the responsibility and power to provide professional accreditation.

We may look for examples in mathematics of the parallel development of the scientific discipline outside the established and accepted model of the profession. One such example is Dirac's delta function which, about 20 years after being in full use among physicists, was expropriated and became a mathematical object, structured by the theory of distributions. This process is an aspect of the internal dynamics of knowledge vis-à-vis society.

There is unquestionably a timelag between the appearance of new ideas in mathematics outside the circle of its practitioners and the recognition of these ideas as "theorizable" into mathematics, endowed with the appropriate codes of the discipline, until the expropriation of the idea and its formalization as mathematics. During this period of time the idea is put to use and practiced: it is an example of what we call ethnomathematics in its broad sense. Eventually it may become mathematics in the style or mode of thought recognized as such. In many cases it never gets formalized, and the practice continues restricted to the culturally differentiated group which originated it. The mechanism of schooling replaces these practices by other equivalent practices which have acquired the status of mathematics, which have been expropriated in their original forms and returned in a codified version.

We claim a status for these practices, ethnomathematics, which do not reach the level of mathematization in the usual, traditional sense. Paraphrasing the terminology of T.S. Kuhn, we say they are not "normal mathematics" and it is very unlikely they will generate "revolutionary mathematics." Ethnomathematics keeps its own life, evolving as a result of societal change, but the new forms simply replace the former ones, which go into oblivion. The cumulative character of this form of knowledge cannot be recognized, and its status as a scientific discipline becomes questionable. The internal revolutions in ethnomathematics, which result from societal changes as a whole, are not sufficiently linked to "normal ethnomathematics". The chain of historical development, which is the spine of a body of knowledge structured as a discipline, is not recognizable. Consequently ethnomathematics is not recognized as a structured body of knowledge, but rather as a set of *ad hoc* practices.

It is the purpose of our research program to identify within ethnomathematics a structured body of knowledge. To achieve this it is essential to follow steps 1, 2, and 3 above.

As things stand now, we are collecting examples and data on the practices of culturally differentiated groups which are identifiable as mathematical practices, hence ethnomathematics, and trying to link these practices into a pattern of reasoning, a mode of thought. Using both cognitive theory and cultural anthropology we hope to trace the origin of these practices. In this way a systematic organization of these practices into a body of knowledge may follow.

IV. Conclusion

For effective educational action not only an intense experience in curriculum development is required, but also investigative and research methods that can absorb and understand ethnomathematics. And this clearly requires the development of quite difficult anthropological research methods relating to mathematics, a field of study as yet poorly cultivated. Together with the social history of mathematics, which aims at understanding the mutual influence of socio-cultural, economic and political factors in the development of mathematics, anthropological mathematics, if we may coin a name for this speciality, is a topic which we believe constitutes an essential research theme in Third World countries, not as a mere academic exercise, as it now draws interest in the developed countries, but as the underlying ground upon which we can develop curriculum in a relevant way.

Curriculum development in Third World countries requires a more global, clearly holistic approach, not only by considering methods, objectives and contents in solidarity, but mainly by incorporating the results of anthropological findings into the 3-dimensional space which we have used to characterize curriculum. This is quite different than what has frequently and mistakenly been done, which is to incorporate these findings individually in each coordinate or component of the curriculum.

This approach has many implications for research priorities in mathematics education for Third World countries and has an obvious counterpart in the development of mathematics as a science. Clearly the distinction between Pure and Applied Mathematics has to be interpreted in a different way. What has been labelled Pure Mathematics, and continues to be called such, is the natural result of the evolution of the discipline within a social, economic and cultural atmosphere which cannot be disengaged from the main expectations of a certain historical moment. It cannot be disregarded that L. Kronecker ("God created the integers — the rest is the work of men"), Karl Marx, and Charles Darwin were contemporaries. Pure Mathematics, as opposed to Mathematics, came into consideration at about the same time, with obvious political and philosophical undertones. For Third World countries this distinction is highly artificial and ideologically dangerous. Clearly, to revise curriculum and research priorities in such a way as to incorporate national development priorities into the scholarly practices which characterizes university research is a most difficult thing to do. But all the difficulties should not disguise the increasing necessity of pooling human resources for the more urgent and immediate goals of our countries.

This poses a practical problem for the development of mathematics and science in Third World countries. The problem leads naturally to a close for the theme of this paper: that is, the relation between science and ideology.

Ideology, implicit in dress, housing, titles, so superbly denounced by Aimé Cesaire in *La Tragédie du Roi Christophe,* takes a more subtle and damaging turn, with even longer and more disrupting effects, when built into the formation of the cadres and intellectual classes of former colonies, which constitute the majority of so-called Third World countries. We should not forget that colonialism grew together in a symbiotic relationship with modern science, in particular with mathematics, and technology.

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