New Parameters for Solution of Two-Well Dispersion Problem

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Abstract: New parameters are presented for the analytical solution of the dispersion problem in two-well injection withdrawal systems. The new expressions are derived to parallel the *a* and *b* parameters presented by Hoopes and Harleman. These two parameters are the dimensionless time required for a tracer to move from the recharge well to any point along a streamline and the spread of the tracer distribution with respect to the mean travel time, respectively. The previously published parameters are found to be correct only in a special case (points on the line connecting the two wells). Validation of the new parameters is compared with a numerical integration scheme using Simpson's rule.

INTRODUCTION

In the last 3 decades, many analytical solutions that could be used to quantify the predict contaminant transport in underground strata were developed for flow fields with variable velocity. Such information is needed for effective applications of remedial technologies to restore polluted aquifers and make them usable again. A system of withdrawal and injection wells is often used as a possible remedial technique to remove contaminated ground water from aquifers and prevent further spreading of contaminants to unpolluted regions. Two-well systems are also employed in tracer tests, where a tracer is injected and its migration in the aquifer is monitored at different observation wells.

In this context, studies in the field of longitudinal and lateral dispersion in two-well flow-through porous media were initiated by Hoopes and Harleman (1967b), who obtained analytical expressions for the temporal and spatial distributions of a dissolved, conservative substance added to the flow between a recharging and a pumping well in a homogeneous, isotropic, and confined aquifer of infinite horizontal extent. A different method was proposed by Grove and Beetem (1971) based on travel time calculations. Analytical solutions to the convective-dispersive transport equation for a single well radial flow were developed by Tang and Babu (1979), Hsieh (1986), and Chen (1987) based on theoretical considerations contributed by Ogata (1958), Lau et al. (1957), Raimondi et al. (1959), and Hoopes and Harleman (1967a).

This study presents new expressions for the parameters obtained in the solution developed by Hoopes and Harleman (1967b) for the dispersion problem in the two-well flow-through porous media. New analytical expressions are developed for the parameters introduced by Hoopes and Harleman (1967b) for additional cases not studied previously. The parameters reported in Hoopes and Harleman (1967b) are of limited use because they only apply correctly to a limited region in the flow domain (points along the *x*-axis). The new parameters proposed in this paper apply to any point in the flow domain and are compared favorably against a numerical solution using Simpson's rule in all cases.

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MATHEMATICAL STATEMENT OF PROBLEM

The general equation describing the distribution of a dissolved substance introduced into a 2D or plane ground-water flow through a homogeneous, isotropic, and porous medium can be written in terms of the potential ϕ and the stream function ψ (Hoopes and Harleman 1967b)

$$\frac{\partial c}{\partial t} + q^2 \frac{\partial c}{\partial \phi} = q^2 \frac{\partial}{\partial \phi}\left((D_1 + D_m)\frac{\partial c}{\partial \phi}\right) + q^2 \frac{\partial}{\partial \psi}\left((D_2 + D_m)\frac{\partial c}{\partial \psi}\right) + S \quad (1)$$

in which c = average concentration of the solute (mass of solute per mass of solution); S = rate of gain of substance within the volume due to leaching from the porous medium or chemical reaction; and D_1, D_2, and D_m = coefficients of longitudinal, lateral dispersion, and molecular diffusion, respectively.

The coefficients of longitudinal dispersion D_1 and lateral dispersion D_2 are defined as $D_1 = \alpha_1 q$ and $D_2 = \alpha_2 q$, respectively, where q is the average seepage velocity along the flow line and α_1 and α_2 are constants, called the intrinsic dispersivity coefficients, and are assumed to be functions of the media structure only.

In the problem under consideration, the aquifer is assumed to be infinite, homogenous, and isotropic and confined between two horizontal planes separated by a vertical distance H. For steady, laminar flow between two wells (one recharging and the other pumping at the same rate), the stream and potential function can be used to map out both the flow and the potential lines, respectively, between the pumping and the recharging well. The stream function ϕ and potential function ψ for such a flow field can be written

$$\frac{\phi}{A} = \tanh^{-1}\left[\frac{2x_0x}{x_0^2 + (x^2 + y^2)}\right] \quad (2)$$

$$\frac{\psi}{A} = \tan^{-1}\left[\frac{2x_0y}{x_0^2 - (x^2 + y^2)}\right] \quad (3)$$

in which $A = Q/2\pi H\theta$, where θ is the aquifer porosity.

Fig. 1 shows on a Cartesian coordinate system the resulting value of the stream and potential functions in the case of the flow field under consideration. Eqs. (2) and (3) allow one to produce (between two wells: one pumping and one recharging) contour lines that are actually a map of the hydraulic head in the aquifer.

The seepage velocity can be expressed (Hoopes and Harleman 1967b)

$$q = \frac{A}{x_0}\left[\cosh\left(\frac{\phi}{A}\right) + \cos\frac{\psi}{A}\right] \quad (4)$$

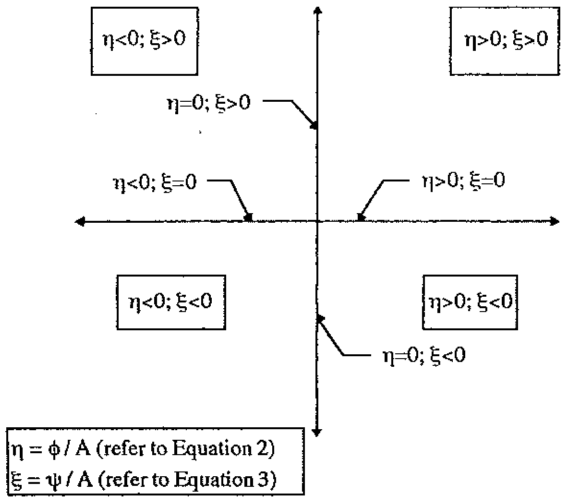

FIG. 1. Values of Potential and Stream Functions for Cases of Recharging and Pumping Wells—Only Cases that Were Studied by Hoopes and Harleman (1967b) Are $\eta < 0$ or $\eta > 0$; $\xi = 0$, for which *a* and *b* Are Correct ($\gamma = 0$), and $\eta < 0$; $\xi < 0$, for which Only *a* Is Correct

In Hoopes and Harleman (1967b) the following four cases were addressed and perspective solutions were derived:

- No dispersion or diffusion along or transverse to the streamlines
- Dispersion and diffusion along the streamlines but no dispersion or diffusion across the streamlines
- Dispersion and diffusion transverse to the streamlines but no dispersion or diffusion along the streamlines
- Combined influences of dispersion, diffusion, and convection

The solution to each case is given in terms of two parameters defined as *a* and *b*, which are the dimensionless time required for a tracer to move from the recharge well to any point along a streamline and the spread of the tracer distribution with respect to the mean travel time, respectively. In this paper, separate generalized solutions for each of these two parameters is proposed. The new parameters could therefore be used in the solutions to the four cases addressed by Hoopes and Harleman (1967b).

The solution to the term *a* (i.e., dimensionless time required for a tracer to move from the recharge well to any point along a streamline) was defined

$$a = \int_{-\infty}^{\eta} \frac{d\eta}{(q')^2} \tag{5}$$

where

$$q' = x_0 q/A = \cosh\eta + \cos\xi, \quad \xi = \psi/A, \ \eta = \phi/A \tag{6}$$

Inserting (6) into (5) yields

$$a = \int_{-\infty}^{\eta} \frac{d\eta}{(\cosh\eta + \cos\xi)^2} \tag{7}$$

Similarly, the term *b* (i.e., the spread of the tracer distribution with respect to the mean travel time) was defined

$$b = \int_{-\infty}^{\eta} \left[\frac{\beta}{(q')^3} + \frac{\gamma}{(q')^4}\right] d\eta \tag{8}$$

where $\beta = \alpha_l/x_0$; and $\gamma = D_m/A$.

Inserting (6) into (8) yields, for the parameter *b*

$$b = \int_{-\infty}^{\eta} \left[\frac{\beta}{(\cosh\eta + \cos\xi)^3} + \frac{\gamma}{(\cosh\eta + \cos\xi)^4}\right] d\eta \tag{9}$$

The nine cases for which parameters *a* and *b* are evaluated in this paper are

- $\eta < 0$; $\xi < 0$
- $\eta < 0$; $\xi = 0$ or 2π
- $\eta < 0$; $\xi > 0$
- $\eta = 0$; $\xi < 0$
- $\eta = 0$; $\xi = 0$ or 2π
- $\eta = 0$; $\xi > 0$
- $\eta > 0$; $\xi < 0$
- $\eta > 0$; $\xi = 0$ or 2π
- $\eta > 0$; $\xi > 0$

The next section presents the approach used to study the integrals of (7) and (9).

EVALUATION OF INTEGRAL

Evaluation of the integrals in (7) and (9) requires one to consider the nine different cases listed above, depending on the sign of the variables ξ and η (or the sign of the stream function ψ and potential function ϕ).

Hoopes and Harleman (1967b) solved the integrals correctly for the parameters *a* and *b* (for $\gamma = 0$) for the case $\eta < 0$ or $\eta > 0$, and $\xi = 0$, 2π (points on the *x*-axis). For the case $\eta < 0$ and $\xi > 0$, they provided an incorrect formula for parameter *b*. They evaluated and plotted the concentration distribution at the withdrawal well and at the midpoint between the two wells with and without dispersion along the streamlines. Breakthrough curves at other points in the flow field were not reported. The solutions provided cannot be computed numerically to redevelop corresponding breakthrough curves at other points within the flow field. No reference to the literature regarding this issue was found, and this suggested that a completely new evaluation of the integrals was necessary. In this paper, the solutions to all nine ξ and η cases shown above are provided (i.e., the nine stream function ψ and potential function ϕ conditions). The new parameters will prove useful as a tool to predict and plot breakthrough curves in practical engineering investigations.

The solutions to each of the nine cases are presented in Table 1. The details of the derivation (Maloof 1998) are available on microfiche for each case of interest.

COMPUTATION OF NEW EXPRESSIONS

For the case of dispersion and diffusion along the streamlines, but no dispersion or diffusion across the streamlines, the concentration at an observation well at any point (*x*,*y*) can be predicted using (Hoopes and Harleman 1967b)

$$\frac{c}{c_0} = \frac{1}{2}\,\mathrm{erf}_c\left[\frac{a - \tau}{\sqrt{4b}}\right] \tag{10}$$

To further investigate the validity of the new parameters obtained in this study, numerical values were calculated for a field situation, using (10). In this scenario (Fig. 2), an aquifer was considered to be confined with two wells, one recharging and one pumping, at 0.02 m^3/s. The distance between the two wells was set as 152 m ($x_0 = 76$ m), and the aquifer had the following properties: thickness $H = 33$ m, $D_m = 0$, $\alpha_l = 1.17$ m, and a porosity of 0.35. In Figs. 3 and 4, and breakthrough curves that show the arrival of the tracer at each of the selected coordinate points are plotted. The dimensionless time $\tau = 1$ corresponds to the actual time of 244 days in Figs. 3 and 4. As can be seen, the new developed parameters predict and

TABLE 1. Solutions to Nine Cases for which Parameters *a* and *b* Are Evaluated

Case (1)	Solution (2)
$\eta < 0; \xi < 0$ $\eta < 0; \xi > 0$	$a = \int_{-\infty}^{\eta} \frac{d\eta}{(q')^2} = \csc^2\xi \{ 1 + \frac{\sinh\eta}{(\cos\xi + \cosh\eta)} - \mathrm{sign}(\xi)\cot\xi (\sin^{-1}\frac{(1 + \cos\xi\cosh\eta)}{(\cos\xi + \cosh\eta)} - \sin^{-1}(\cos\xi)) \}$ $b = \int_{-\infty}^{\eta} [\frac{\beta}{(\cosh\eta + \cos\xi)^3} + \frac{\gamma}{(\cosh\eta + \cos\xi)^4}] d\eta = \frac{\beta}{2\sin^2\xi} [\frac{\sinh\eta}{(\cos\xi + \cosh\eta)^2} - \frac{3\cos\xi}{\sin^2\xi} (\frac{\sinh\eta}{(\cos\xi + \cosh\eta)}$ $+ 1 + \mathrm{sign}(\xi)\frac{2}{3}\cot\xi\sin^{-1}(\cos\xi) - \mathrm{sign}(\xi)\frac{2}{3}\cot\xi\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{(\cos\xi + \cosh\eta)}) - \mathrm{sign}(\xi)\frac{1}{\sin^3\xi}$ $\cdot \{ -\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{(\cos\xi + \cosh\eta)} + \sin^{-1}(\cos\xi) \}] - \frac{\gamma}{3\sin^2\xi} [-\frac{\sinh\eta}{(\cos\xi + \cosh\eta)^3} - \frac{2}{\sin^4\xi} \{ \frac{\sinh\eta}{(\cos\xi + \cosh\eta)}$ $+ 1 + \mathrm{sign}(\xi)\frac{9}{4}\cot\xi\sin^{-1}(\cos\xi) - \mathrm{sign}(\xi)\frac{9}{4}\cot\xi\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{\cos\xi + \cosh\eta} \} + \frac{5\cos\xi}{2\sin^2\xi} \langle \frac{\sinh\eta}{(\cos\xi + \cosh\eta)^2} + \frac{3\cos\xi}{2\sin^2\xi}$ $\cdot \{ -\frac{22}{15}\frac{\sinh\eta}{(\cos\xi + \cosh\eta)} - \frac{22}{15} - \mathrm{sign}(\xi)\frac{4}{5}\cot\xi\sin^{-1}(\cos\xi) + \mathrm{sign}(\xi)\frac{4}{5}\cot\xi\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{\cos\xi + \cosh\eta} \} \rangle]$
$\eta < 0; \xi = 0$ or 2π $\eta > 0; \xi = 0$ or 2π $\eta = 0; \xi = 0$	$a = \int_{-\infty}^{\eta} \frac{d\eta}{(q')^2} = \frac{1}{3} \{ 1 + \tanh(\frac{\eta}{2}) [1 + \frac{1}{(1 + \cosh\eta)}] \}$ $b = \int_{-\infty}^{\eta} [\frac{\beta}{(\cosh\eta + \cos\xi)^3} + \frac{\gamma}{(\cosh\eta + \cos\xi)^4}] d\eta = \frac{2\beta}{(3)(5)} (\frac{\sinh\eta}{(1 + \cosh\eta)} [\frac{3}{2}\frac{1}{(1 + \cosh\eta)^2} + \frac{1}{(1 + \cosh\eta)} + 1] + 1)$ $+ \frac{2\gamma}{(5)(7)} (1 + \frac{\sinh\eta}{(1 + \cosh\eta)} [\frac{5}{2}\frac{1}{(1 + \cosh\eta)^3} + \frac{3}{2}\frac{1}{(1 + \cosh\eta)^2} + \frac{1}{(1 + \cosh\eta)} + 1])$
$\eta > 0; \xi < 0$ $\eta > 0; \xi > 0$ $\eta = 0; \xi < 0$ $\eta = 0; \xi > 0$	$a = \int_{-\infty}^{\eta} \frac{d\eta}{(q')^2} = 2 [\csc^2\xi \{ 1 + \mathrm{sign}(\xi)\cot\xi (\sin^{-1}(\cos\xi) - \frac{\pi}{2}) \}] - [\csc^2\xi \{ 1 - \frac{\sinh\eta}{(\cos\xi + \cosh\eta)} - \mathrm{sign}(\xi)\cot\xi$ $\cdot (-\sin^{-1}(\cos\xi) + \sin^{-1}\frac{(1 + \cos\xi\cosh\eta)}{(\cos\xi + \cosh\eta)}) \}]$ $b = \int_{-\infty}^{\eta} [\frac{\beta}{(\cosh\eta + \cos\xi)^3} + \frac{\gamma}{(\cosh\eta + \cos\xi)^4}] d\eta = \frac{\beta}{\sin^2\xi} [-\frac{3\cos\xi}{\sin^2\xi} (1 - \mathrm{sign}(\xi)\frac{2}{3}\cot\xi\frac{\pi}{2} + \mathrm{sign}(\xi)\frac{2}{3}\cot\xi\sin^{-1}(\cos\xi))$ $- \mathrm{sign}(\xi)\frac{1}{\sin^3\xi} \{ +\sin^{-1}(\cos\xi) - \frac{\pi}{2} \} - \frac{2\gamma}{3\sin^2\xi} [-\frac{2}{\sin^4\xi} \{ 1 + \mathrm{sign}(\xi)\frac{9}{4}\cot\xi\sin^{-1}(\cos\xi) - \mathrm{sign}(\xi)\frac{9}{4}\cot\xi\frac{\pi}{2} \}$ $+ \frac{5\cos\xi}{2\sin^2\xi} \langle \frac{3\cos\xi}{2\sin^2\xi} \{ -\frac{22}{15} - \mathrm{sign}(\xi)\frac{4}{5}\cot\xi\sin^{-1}(\cos\xi) + \mathrm{sign}(\xi)\frac{4}{5}\cot\xi\frac{\pi}{2} \} \rangle]]$ $- [\frac{\beta}{2\sin^2\xi} [\frac{-\sinh\eta}{(\cos\xi + \cosh\eta)^2} - \frac{3\cos\xi}{\sin^2\xi} (\frac{-\sinh\eta}{(\cos\xi + \cosh\eta)} + 1 + \mathrm{sign}(\xi)\frac{2}{3}\cot\xi\sin^{-1}(\cos\xi)$ $- \mathrm{sign}(\xi)\frac{2}{3}\cot\xi\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{(\cos\xi + \cosh\eta)}) - \mathrm{sign}(\xi)\frac{1}{\sin^3\xi} \{ -\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{(\cos\xi + \cosh\eta)} + \sin^{-1}(\cos\xi) \}]$ $- \frac{-\gamma}{3\sin^2\xi} [\frac{\sinh\eta}{(\cos\xi + \cosh\eta)^3} - \frac{2}{\sin^4\xi} \{ \frac{-\sinh\eta}{(\cos\xi + \cosh\eta)} + 1 + \mathrm{sign}(\xi)\frac{9}{4}\cot\xi\sin^{-1}(\cos\xi)$ $- \mathrm{sign}(\xi)\frac{9}{4}\cot\xi\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{\cos\xi + \cosh\eta} \} + \frac{5\cos\xi}{2\sin^2\xi} \langle \frac{-\sinh\eta}{(\cos\xi + \cosh\eta)^2} + \frac{3\cos\xi}{2\sin^2\xi} \{ +\frac{22}{15}\frac{\sinh\eta}{(\cos\xi + \cosh\eta)}$ $- \frac{22}{15} - \mathrm{sign}(\xi)\frac{4}{5}\cot\xi\sin^{-1}(\cos\xi) + \mathrm{sign}(\xi)\frac{4}{5}\cot\xi\sin^{-1}\frac{1 + \cos\xi\cosh\eta}{\cos\xi + \cosh\eta} \} \rangle]]$

Note: sign(ξ) = 1 for $\varepsilon > 0$ and -1 for $\xi < 0$.

describe in a meaningful way the expected tracer concentration at the designated coordinates.

COMPARISONS BETWEEN SOLUTIONS

To verify the analytical solution, the new parameters (a and b) were also computed by numerical integration using Simpson's rule. In this scenario, numerical values were calculated for a sandbox laboratory experiment using (10), with two wells, one recharging and one pumping, at 100 cm^3/s. The distance between the two wells was set as 200 cm (x_0 = 100 cm), and the aquifer had the following properties: thickness H = 500 cm and a porosity of 0.20. As shown in Table 2, the terms used in the solutions are found to be equivalent. This finding confirms the appropriateness of the developed parameters as a new tool to be used in the approximate analytical solution developed by Hoopes and Harleman (1967b) [(7) and (9)] to predict tracer movement between a pumping and a recharging well.

The new developed parameters can be readity calculated in spreadsheet format (available from the writers) and used to obtain breakthrough curves at any point within a given flow field for a variety of problems.

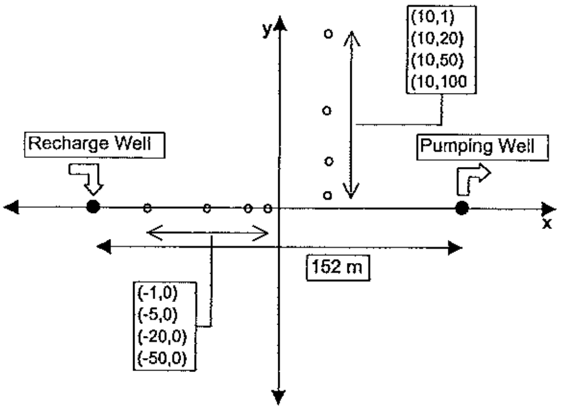

FIG. 2. Schematic of Field Situation Considered

TABLE 2. Results Comparison between Numerical Method Using Simpson's Rule and Analytical Solution

Case (1)	X (2)	Y (3)	ξ (4)	η (5)	Numerical solution (6)	Analytical solution (7)
I/q'^2	−10	10	0.20132	−0.19865	0.294581	0.288443
	−10	−10	−0.20132	−0.19865	0.294581	0.288443
	10	−10	−0.20132	0.19865	0.395502	0.389136
	10	10	0.20132	0.19865	0.395502	0.389136
I/q'^3	−10	10	0.20132	−0.19865	0.114585	0.111512
	−10	−10	−0.20132	−0.19865	0.114585	0.111512
	10	−10	−0.20132	0.19865	0.165386	0.162205
	10	10	0.20132	0.19865	0.165386	0.162205
I/q'^4	−10	10	0.20132	−0.19865	0.04802	0.046482
	−10	−10	−0.20132	−0.19865	0.04802	0.046482
	10	−10	−0.20132	0.19865	0.073592	0.072003
	10	10	0.02132	0.19865	0.073592	0.072003

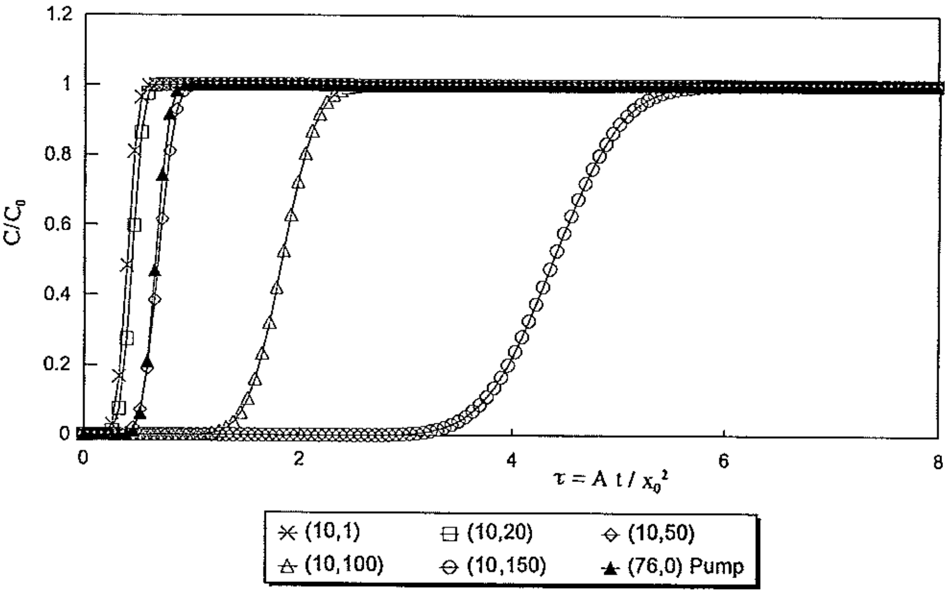

FIG. 3. Plot of Concentration Distribution along $\xi > 0$ and $\eta > 0$

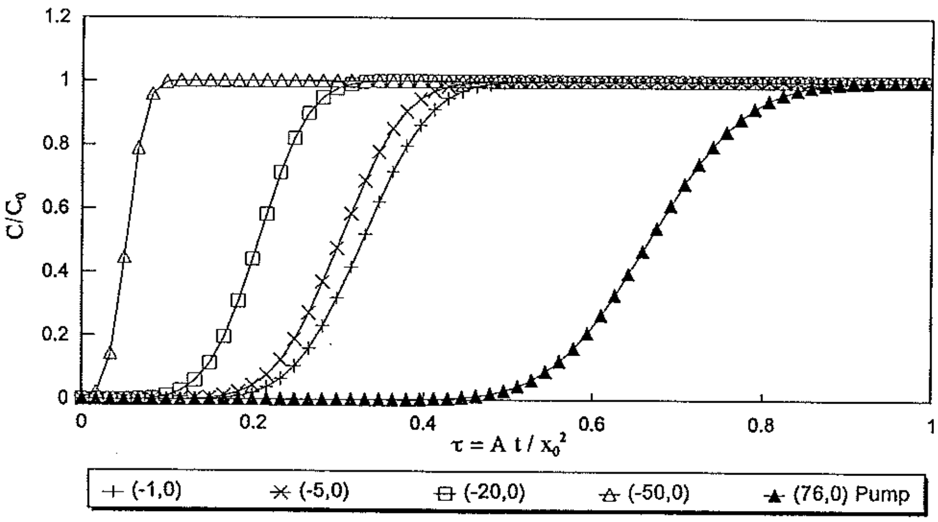

FIG. 4. Plot of Concentration Distribution along $\xi = 0$ and $\eta < 0$

SUMMARY AND CONCLUSIONS

In this paper new parameters are presented for the analytical solution of the dispersion problem in two-well injection withdrawal systems. The new expressions are derived to parallel the a and b parameters presented by Hoopes and Harleman (1967b). The new parameters are comparable to the parameters developed by Hoopes and Harleman (1967b) only for points on the x-axis ($\xi = 0$ or 2π). The parameters are compared against a numerical solution using Simpson's rule and are found to be equivalent.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = $Q/2\pi H\theta$ (L^2/T);
a = travel time from recharge well to any point along streamline (T);
b = measure of variance of tracer distribution (T^2);
c = concentration of tracer, mass of tracer to mass of solution;
c_0 = tracer concentration at recharge well;
D_m = molecular diffusion coefficient (L^2/T);
D_1, D_2 = coefficients of longitudinal and lateral dispersions, respectively, in uniform flow (L^2/T);
$\text{erf}(\Delta)$ = error function of $\Delta = (2/\sqrt{\pi}) \int_0^\Delta e^{-\lambda^2} d\lambda$;
$\text{erfc}(\Delta)$ = complementary error function of $\Delta = 1 - \text{erf}(\Delta)$;
H = thickness of confined aquifer (L);
h = piezometric head (L);
n = coordinate along equipotential line (L);
Q = well flow rate (L^3/T);
q = seepage velocity along streamline (L/T);
q' = $x_0 q/A$;
s = coordinate along streamline (L);
t = time (T);
x_0 = one-half well spacing (L);
α_1, α_2 = longitudinal and lateral dispersivity coefficients, respectively, in nonuniform flow (L);
θ = porosity;
τ = dimensionless time coordinate = At/x_0^2;
ϕ = equipotential function (L^2/T); and
ψ = stream function (L^2/T).