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# Progress on the Theory of Flow in Geologic Media with Threshold Gradient

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**Abstract**

Several experimental studies have indicated that the traditional Darcy law is not valid at low filtration velocities. These results do not represent rare exceptions, but rather the ordinary behavior of certain fluid-porous media geosystems, such as water and gas flowing through soils containing wet clays. In addition, viscoplastic fluids, such as oils containing paraffin, asphaltene and wax, flowing under low temperature in porous media exhibit similar deviations from the Darcian behavior. The common conclusion of these studies, which are summarized in the first part of this paper, is that non-Darcian fluid flow, in many cases, may be described by a filtration law with threshold gradient: there is no flow for forced pressure gradient values below the threshold gradient and there is a linear relationship between flow velocity and pressure gradient above the threshold gradient value.

The second part of this paper deals with the theoretical aspects of fluid flow with threshold gradient. Solutions to several two-dimensional steady-state horizontal flow problems are discussed. The solutions show that the typical characteristic of the flow regime is stagnation zones which appear in the vicinity of the critical points of the flow. When water is injected into the reservoir to displace the non-Newtonian fluid, the stagnation zones change and achieve a final limiting configuration. The size of the stagnation zones at that time determines the loss of the displaced

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fluid, which may be minimized by increasing the flow rate of the displacing liquid (water).

The last section of this paper deals with nonsteady flow situations and introduces methods to estimate field scale reservoir parameters, particularly the field scale threshold gradient. A number of publications available only in Russian are critically presented and compared to the western literature.

## Introduction

The theory of flow in porous media has been developed to address technical problems in the fields of civil, environmental, and petroleum engineering. For example, the theory is applied in hydrogeologic investigations of natural groundwater flow and drainage in a variety of geologic formations. Another application is the replenishment of underground reservoirs by various artificial recharge methods, such as surface spreading techniques or deep injection through wells. Of special interest are the cases of sewage or liquid radioactive waste disposal into deep formations. In the field of soil mechanics, consolidation studies, especially for clay soils, require flow modeling.

The theory has also been used to predict contaminant migration in the subsurface environment. Accidental releases of hazardous fluids due to oil and chemical spills and leaks from underground petroleum storage tanks are a great national concern because of the associated risks of aquifer contamination. Use of the theory is essential to predict contaminant transport through soils and to select appropriate remedial actions. Flow in low permeability geologic media affects the geochemical evolution of groundwater systems over geologic times and leads to the formation of mineral deposits.

Simultaneous flow of oil and water is encountered in practically all oil reservoirs during the oil production process. Of special interest are methods of water or chemical (aqueous surfactants, polymers) flooding in which the displacing liquid recovers oil by invading the oil reservoir during secondary recovery operations.

Numerous observations from engineering practice and laboratory studies suggest that, in certain cases, it is necessary to reconsider the established fundamentals of the theory of flow in porous media and to develop alternative hydrodynamic models. Such reexamination of those fundamentals may well result in the discovery of how different physical behaviors of the fluid-geologic medium system affect the mentioned technical applications.



The theory of non-Newtonian flow in geologic media provides an example. Studies of non-Newtonian flow in porous media were initiated when experimental results indicated that the traditional Darcy's law was not valid at low filtration velocities. Flow with a threshold gradient is a special case: no flow occurs unless the gradient exceeds a limiting value. In due course it became evident that such results were not exceptions, but rather the ordinary behavior of certain fluid-porous media systems, such as water and gas flowing through argillaceous soils. In addition, viscoplastic fluids, such as oils containing paraffin, asphaltene and wax, flowing under low temperature in porous media exhibit similar deviations from the Darcian behavior. The overwhelming experimental evidence left no doubt that certain flows in porous media could not be reliably described by Darcy's theory. Therefore, a solution to such problems was not possible by introducing some correction factors into the existing results, but rather necessitated the development of a hydrodynamic theory of flow with a threshold gradient in geologic media.

Due to its importance to the oil production industry in the former Soviet Union, non-Darcian flow theory in geologic media has been studied extensively by Soviet scientists since the 1950's. Significant progress has been achieved in most of the practical problems, such as unsteady one-phase flow, multidimensional one-phase flow, two-phase flow, well data analysis and other applications of the theoretical results. Numerous publications on these problems exist in Russian, which have not been accessible to interested western scientists due to language barriers. The present review aims at serving this need and at comparing international scientific progress on this subject.

Among the various non-Darcian flow with threshold gradient problems, one-dimensional two-phase flow has been studied well, in part due to the relatively simple theory needed for such studies [3,4,6–9,18,42–49].

In contrast to Darcy's flow, multidimensional flow with a threshold gradient is characterized by zones where the forced pressure gradient is so low that the fluid in these zones becomes immobile (stagnation zones). Although knowledge of the location and geometry of these zones is of paramount interest, practical calculation is faced with tremendous mathematical obstacles. Development of a general theory and calculation methods for the stagnation zones have been pioneered by V.M. Entov and published in two monographs in Russian [7, 21]. A third problem, namely unsteady non-Darcian flow and estimation of field-scale parameters, has also been developed and published in Russian [7, 20].

The main goal of this paper is to review the empirical results available today, which comprise the basis of the theory, and present some available analytical solutions to flow problems, providing the features of multidimensional and unsteady non-Darcian fluid flow in geological size formations, such as water and oil reservoirs. The next section, Experimental Studies of Non-Darcian Behavior, summarizes the experimental evidence for a threshold gradient and introduces an appropriate dynamic law to model such fluid-porous medium systems. The third section presents and discusses solutions to two-dimensional steady-state horizontal flow problems. The fourth section addresses the problem of residual stagnation zones after the displacement of a non-Newtonian fluid by a Newtonian one. The fifth section deals with non-steady flow situations and introduces methods to estimate field scale reservoir parameters. The conclusions and recommendations of this review are summarized in the last section.

## Experimental Studies of Non-Darcian Behavior

The purpose of this section is to present the experimental evidence of threshold gradient flow in low permeability porous media and to summarize explanations of such behavior as proposed in the literature. However, no attempt is made to comprehensively review the literature on microstructure modifications and electrochemical phenomena at microscale, which may explain convincingly the macrobehavior of the fluid-porous medium system.

### Water and Gas Flow in Argillaceous Porous Media

Non-Darcian flow phenomena were repeatedly observed for water moving through porous media containing clay [24, 51, 52]. A typical dependence of water filtration velocity on forced pressure gradient inside an argillaceous porous medium is shown in Figure 1 [11]. The experimental curve is matched with a filtration law with a threshold gradient, which must be exceeded before flow starts.

Similar data have been obtained for gas flow in porous media containing clay and residual water. Typical experimental data obtained in artificial porous media made of a mixture of sand with clay are shown in Figure 2 [1]. These results demonstrate no gas movement if the forced pressure difference



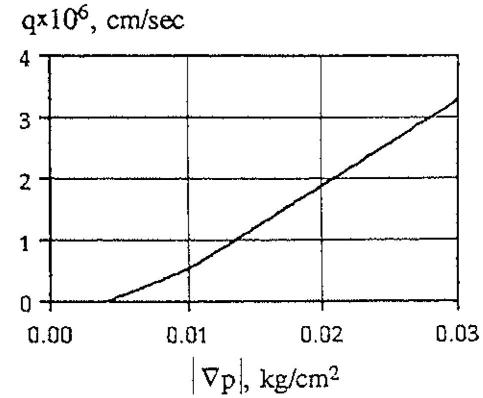


Figure 1 Characteristic curve of water flow through clay (from *Bondarenko and Nerpin*, 1965)

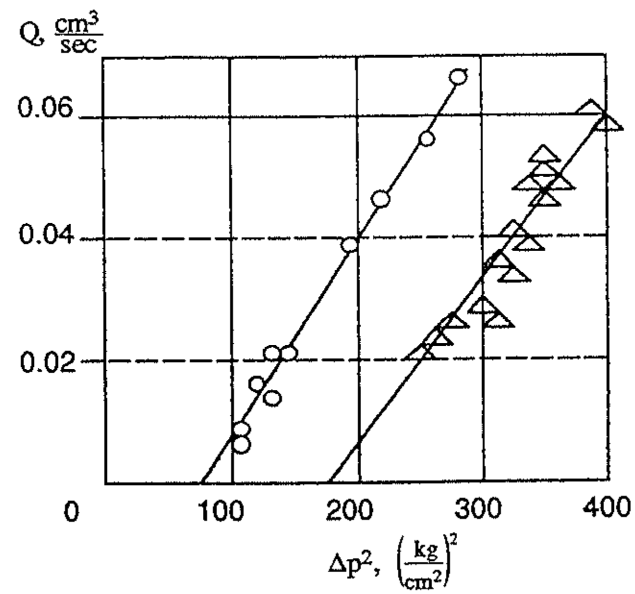


Figure 2 o – porous medium containing 75% sand and 25% clay; Δ – porous medium containing 70% sand and 30% clay; Gas flow in argillaceous porous medium containing 40% residual water (from *Akhmentov et al.*, 1969)

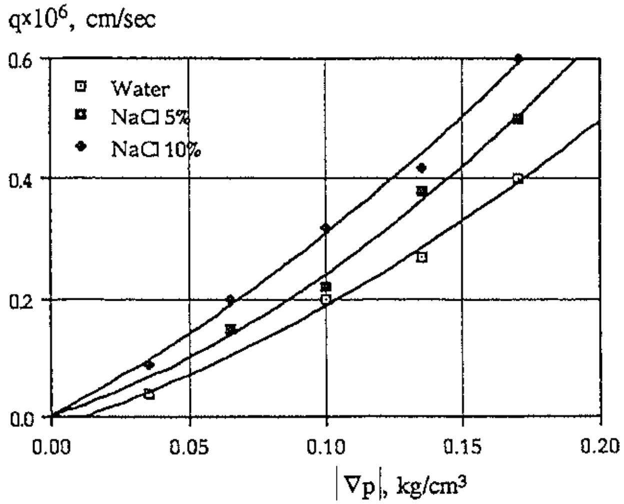


Figure 3 Effect of electrolyte amount on water flow in argillaceous porous media (from *Von Engelhardt and Tunn*, 1954)

is less than some limiting value, which depends on the fraction of clay in the mixture. If the pressure difference exceeds the limiting value the gas flow rate is proportional to the square of the pressure difference.

In fact, the nonlinear effects in all the above cases are related to the interaction between water and clay particles. Polar water molecules interact with clay grains. Due to the small size of the clay grains, near 10 $\mu$, the contact surface between clay and water is very large and may, in some cases, reach several hundred square meters (for example 700 $m^2$/gr for montmorillonite). Therefore, the clay fines are able to absorb a large amount of water and to swell significantly. Furthermore, the clay fines may also tear off the solid matrix creating a gel-forming colloid. Thus, two phenomena occur in the porous media containing clay: (1) water adsorption by the clay fines, and (2) pore clogging with the colloid. Both phenomena cause a decrease in permeability and a nonlinear pseudoplastic fluid behavior at low pressure gradient values. The nonlinear effect depends on the specific conditions and the threshold gradient may or may not be observed [28, 29, 31, 33, 34, 38, 40, 41, 58]. For example, Figure 3 [58] demonstrates the dependence of flow velocity of



an electrolyte solution (NaCl/water) through natural sandstone cores of high permeability (0.2–1.3 Darcies) and low clay content. It can be observed that the nonlinear effect constantly decreases with an increasing concentration of NaCl. In his review of groundwater flow in low-permeability environments, Neuzil [39] points out that there is no consistency in the types and magnitudes of deviations from Darcy's law and that the most carefully conducted experiments are of Darcy-type. He cautions, however, that there is a lack of experimental data for flow in tight media under realistically small hydraulic gradients, and, due to this observational gap, Darcy's equation is just a highly speculative assumption.

In summary, the type of chemicals dissolved in water and the type and percentage of clay fines affect the water flow through argillaceous porous media. Clays are a natural ion exchange media, releasing from their surface metal ions into the solution and, consequently, obtaining a certain surface charge upon interaction with ions in the solution and with polar water molecules. Hence, the electrokinetic effects may alter significantly the resistance of water flow in natural porous media containing clay. Detailed analyses of changes in water properties near solid boundaries and interfaces are given by Low [32], Mitchell [37], Forslind and Jacobson [26], Clifford [12] and most recently by Vaidya [56] and Zhao et al. [63].

## Crude Oil Flow Through Porous Media

Many crude oils contain heavy components (tar, paraffin, asphaltene, wax), which under certain conditions, such as low temperature, may create a solid-like structure within the liquid phase. As a result, the rheological behavior of these gel-forming oils is described by a non-Newtonian law, mostly of a pseudoplastic or viscoplastic type. Nonlinear flow of gel-forming oils through capillaries and porous media has been studied and published extensively [2, 14, 15, 22, 23, 30, 53, 55]. As an example, a nonlinear relation between filtration velocity and forced pressure gradient in artificial porous media made of loose sand is shown in Figure 4 [2]. It can be observed that a threshold pressure gradient exists if oil flows at low temperatures. As temperature increases the threshold gradient decreases until it becomes zero. Figure 5 [25] shows that the porous media permeability also affects the threshold gradient value. Low permeability may drastically increase the threshold gradient.

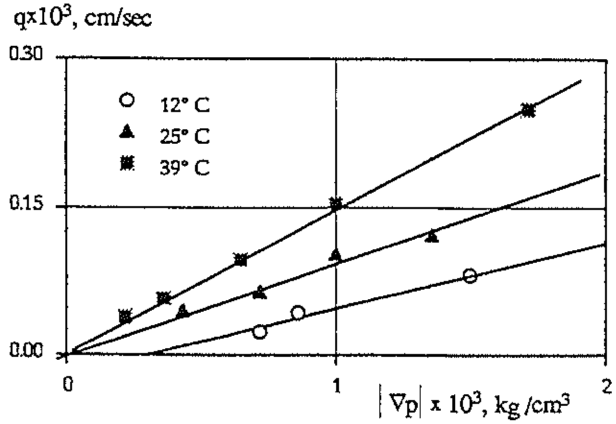


Figure 4 Temperature effect on heavy oil flow through loose sand sample (from *Alishaev et al.*, 1966)

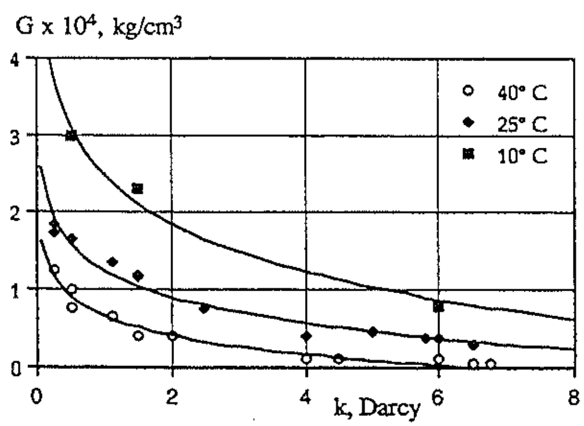


Figure 5 Temperature and permeability effect on threshold gradient (from *Fomenko*, 1968)



## Summary

Numerous experimental results demonstrate that the fundamental Darcy's law, which was established in 1856 and suggested a linear relation between the pressure gradient, $\nabla p$, and the vector of filtration velocity in porous media, $q$, is not valid in many cases. There are two main reasons for this phenomenon: (1) the nonlinear interaction between the fluid and the solid matrix of the porous media followed by a change of the fluid properties inside the thin layer near the solid surface and by the appearance of additional resistance to the flow, especially at low flow velocities, and (2) the non-Newtonian properties of the fluid, which does not obey a model of linearly-viscous flow. In view of these results, alternative dynamic equations were proposed based on the best fit to the experimental data [17, 27, 54]. .

As many experiments demonstrate, non-Darcian fluid flow may be described well by a dynamic law with a threshold gradient of magnitude $G$: there is no flow for forced pressure gradient values below the threshold gradient and a linear relationship between flow velocity and pressure gradient exists above the threshold gradient.

$$q = \begin{cases} -\frac{k}{\mu}\left(\nabla p - G\frac{\nabla p}{|\nabla p|}\right) & \text{for} \quad |\nabla p| > G \\ 0 & \text{for} \quad |\nabla p| \leq G \end{cases} \tag{1}$$

where $k$ is the porous medium permeability and $\mu$ is the fluid viscosity.

It has been established [35, 36] that the threshold gradient $G$ is inversely proportional to the square root of permeability $k$

$$G \approx 1/\sqrt{k} \tag{2}$$

The same relation was later obtained using similarity theory [7]. Experimental verification of the relation was provided by Fomenko [25] (see Figure 5). Recently Vradis and Protopapas [60], using two different microscale models (capillary tube and spherical grain representation of the medium), showed that eq 1 is a valid first approximation, and they also derived eq 2. The dynamic law with a threshold gradient has been used successfully to represent the flow of many fluids in porous media (heavy crudes, water, gas).

There have been only a few publications on large-scale non-Darcian flow in geologic formations using the threshold gradient law (eq 1) in the western literature [10,16,42–49,61]. Volker [57] deals with nonlinear Darcy seepage. Vongvuthipornchai and Raghavan [59] present methods to analyze oil well data for power-law fluids. Yoder and Dube [62] report on generic non-Darcy phenomena during an irrigation experiment. The large number of articles dealing with flow and heat transfer of viscoplastic fluids in pipes, channels, reactors, vessels, etc. are not reviewed in this study, which focuses on flow in geologic media.

## Two-Dimensional Steady Flow

The main characteristic of two-dimensional horizontal flow with a threshold gradient is stagnation zones which can occur in the neighborhood of the critical points of the flow. The shape and size of the stagnation zones as well as their location on the flow domain are of great interest in many applications, such as heavy oil recovery and wastewater disposal.

Desaulniers et al. [13] provide indirect evidence of the existence of the stagnant zones in natural geologic formations. The authors measured $^{35}Cl$ and $^{37}Cl$ concentrations in groundwater flow and compared these results to predictions using Darcy's law. Deviations of the field results from the predictions were explained by the occurrence of stagnation zones due to threshold gradient. However, no direct field or laboratory evidence provides proof that these stagnation zones exist.

A method to derive analytical solutions for two-dimensional steady-state fluid flow with a threshold gradient in homogeneous porous media has been developed by V. M. Entov and has been thoroughly studied in Bernadiner and Entov [7]. Introducing a characteristic velocity, $\lambda = kG/\mu$, and taking into account the collinearity of pressure gradient, $\nabla p$, and filtration velocity vector, $q$, the basic equations 1 are transformed into the system

$$\frac{k}{\mu}\nabla p \begin{cases} = -\left[1 + \frac{\lambda}{|q|}\right] q & |q| > 0 \\ \geq \lambda & |q| > 0 \end{cases} \tag{3}$$

The continuity equation is div $q = 0$.

An exact solution to the horizontal flow problem governed by equation system 3, in many cases of symmetrical flow, can be achieved by introducing the stream function, $\psi$, in terms of the hodograph variables ($\omega = |q|$ and $\theta$, where $\theta$ is the angle of the flow velocity vector with the $x$ axis). The stream function, $\psi$, is related to the pressure, $p$, through the following equations:

$$\frac{(\omega + \lambda)^2}{\omega}\frac{\partial \psi}{\partial \omega} = \frac{k}{\mu}\frac{\partial p}{\partial \theta} \qquad \frac{\omega + \lambda}{\omega^2}\frac{\partial \psi}{\partial \theta} = \frac{k}{\mu}\frac{\partial p}{\partial \omega}$$



Then it can be shown that the stream function, $\psi$, satisfies the hypergeometric equation

$$\omega(\omega + \lambda)\psi_{\omega\omega} + (\omega - \lambda)\psi_{\omega} + \psi_{\theta\theta} = 0 \tag{4}$$

The solution to this equation was found by using hypergeometric transformation with respect to variable $\omega$. To convert the solution from the radial hodograph plane $(\omega, \theta)$ to physical plane $(x, y)$, the following integrals must be used:

$$\begin{aligned} x &= -\int \frac{\cos\theta}{\omega + \lambda}\,\frac{k}{\mu}\,dp + \frac{\sin\theta}{\omega}\,d\psi \\ y &= -\int \frac{\sin\theta}{\omega + \lambda}\,\frac{k}{\mu}\,dp - \frac{\cos\theta}{\omega}\,d\psi \end{aligned} \tag{5}$$

The stagnation zone boundaries can be obtained from eq 5 at $\omega = 0$ (specific discharge $q$ equal to zero).

For an infinite number of wells equally spaced at distance $L$ on a straight line, each pumping at equal rate, $Q$, from an unbounded geologic reservoir, the boundary of the stagnation zone is shown in Figure 6 as it changes with the well productivity factor, $b = Q/(4\lambda L)$. It can be seen that decreasing the well productivity factor (lowering the pumping rate or spacing wider) significantly increases the stagnation zones and, consequently, the amount of immovable fluid.

In fact, this is a common conclusion for all of the studied cases of two-dimensional horizontal flow with a threshold gradient. For example, Figure 7 shows the stagnation zone boundaries if the pumping wells in the chain have vertical parallel fractures, each with length $2L$ (twice the distance between the wells). It can be observed that if the characteristic well productivity parameter, $b$, is very low, say, 0.1, the stagnation zone covers almost the entire fracture length.

If non-Darcian flow occurs in a reservoir with a step-like impermeable boundary, the boundary corner, which is a critical point of the flow, is covered by a zone of immovable fluid as shown in Figure 8. The size and shape of the stagnation zone again depends on the characteristic parameter, $b$; the zone area decreases as $b$ increases. In this case, the characteristic distance, $L$, is the step size.

Figure 9 exhibits flow with a threshold gradient through a pumping well chain placed near an impermeable reservoir boundary. It can be seen that even when $b$ is quite high, equal to 1, the boundary is completely covered by immovable fluid.

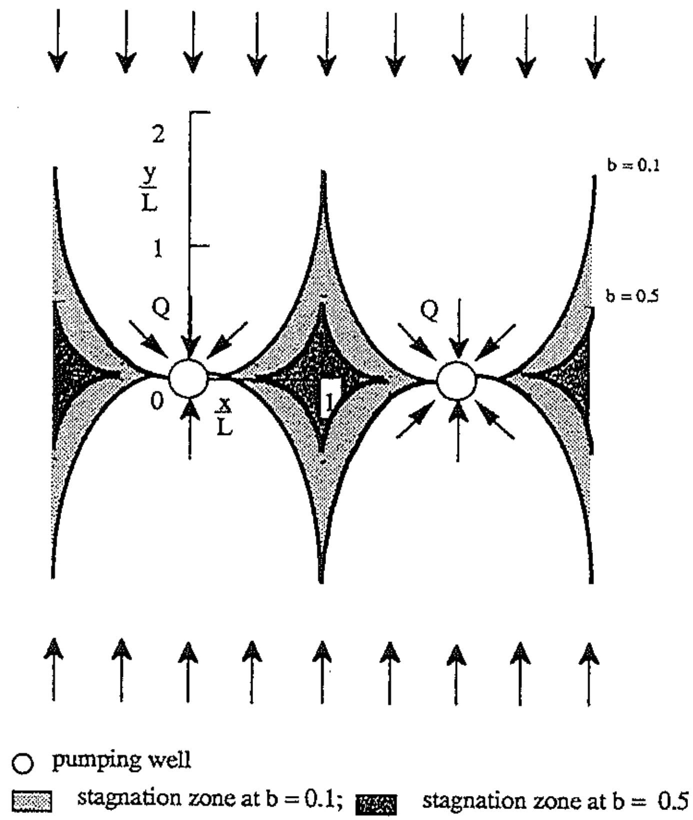


Figure 6 Boundaries of the stagnation zones for a chain of pumping wells (from *Bernadiner and Entov*, 1975)

## Limiting Configuration of the Stagnation Zones

Consider the problem of injection of a Newtonian fluid in a flow domain with stagnation zones as discussed in the previous section. During the displacement of the fluid with a threshold gradient $G > 0$ by a Newtonian liquid, such as water, the stagnation zones, which appear in the neighborhood of critical points of the flow, may change due to changes of the pressure distribution



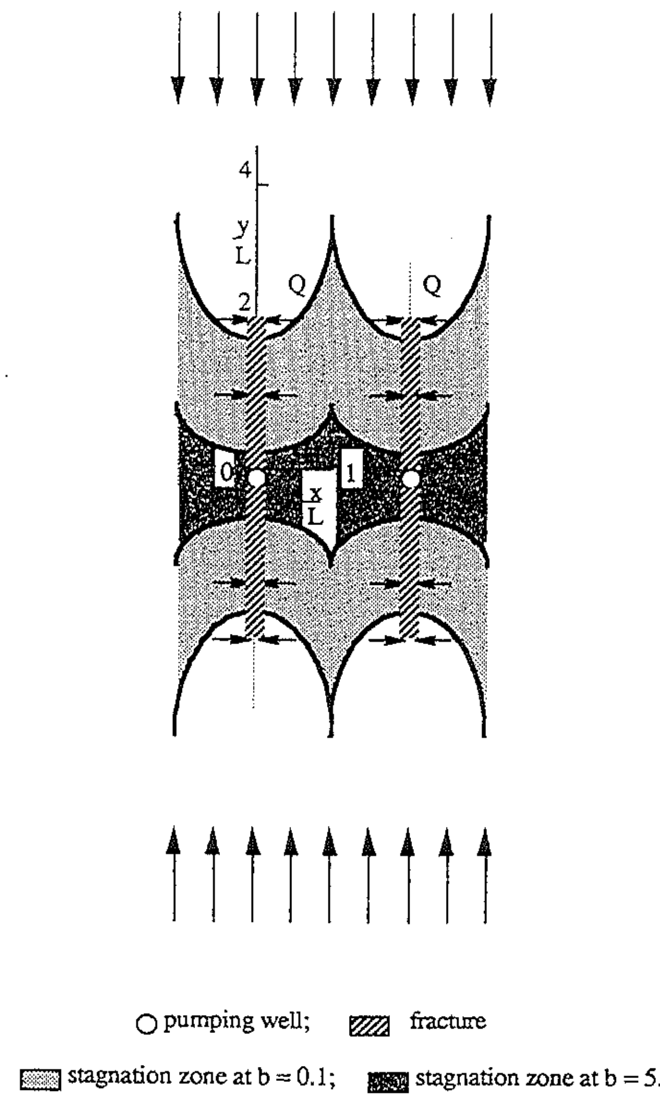


Figure 7

Boundaries of the stagnation zones for a chain of pumping wells with fractures (from *Bernadiner and Entov*, 1975)

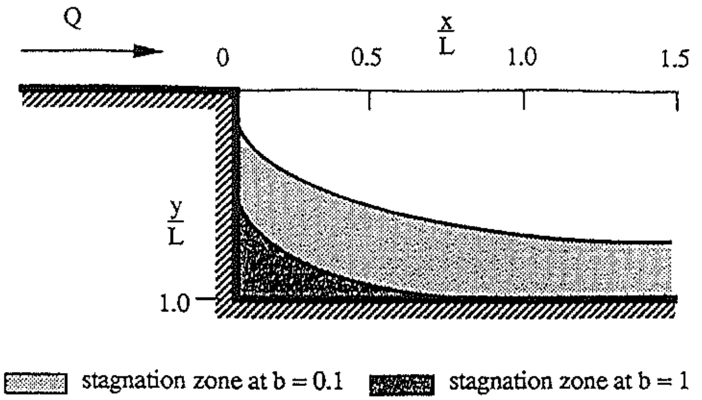


Figure 8

Flow in reservoir with step–like impermeable boundary (from *Bernadiner and Entov*, 1975)

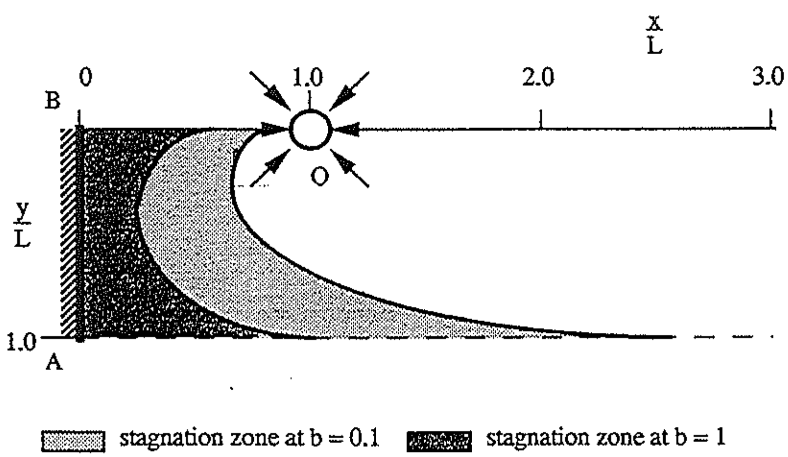


Figure 9

Boundaries of the stagnation zones for a chain of pumping wells near an impermeable boundary (from *Bernadiner and Entov*, 1975)



in the reservoir. The final configuration of the stagnation zones, which determines the amount of non-Newtonian fluid that cannot be recovered, generally depends on the history of the displacement. However, it is possible to find the maximum size of the permanent stagnation zones at the end of the displacement process under constant hydrodynamic conditions. Experimental results of the limiting stagnation zones formed after displacement of viscoplastic crude oil by water in Hele-Shaw cells are reported in Bernadiner and Entov [7]. In the region of the Newtonian fluid (water) Darcy's law is obeyed and the pressure satisfies the Laplace equation

$$q = -\frac{k}{\mu}\nabla p \quad \nabla^2 p = 0 \tag{6}$$

The unknown boundary of the final stagnation zones is a streamline for water, and the value of the pressure gradient on this streamline is equal to the threshold gradient of the displaced fluid.

$$|\nabla p| = G \quad \text{on the unknown stagnation zone boundary} \tag{7}$$

In fact, eq 7 is an additional condition for the boundary determination. Some exact solutions for the system of equations eqs 6 and 7 for symmetrical well patterns in homogeneous porous media are available [5, 7, 21]. The method of solution is based on sequential application of the hodograph transformation and complex variable function theory. Figure 10 (a) illustrates a five-spot well pattern with $2L$ distance between injecting and pumping wells. Figure 10 (b) [5] demonstrates variation of the sweep efficiency $\beta$ (defined as the ratio of the swept area over the total area) with the dimensionless parameter $b = Q/(\lambda L)$. Increase of the injection flow rate, $Q$, or decrease of the well spacing, $L$, results in an increase in sweep efficiency, $\beta$.

A similar analytical method has been employed by Boast and Baveye [10] to obtain the configuration of stagnation zone at the corner problem. The authors approximated Mitchell and Younger's [38] data of flow rate versus pressure gradient with a step-like flow equation, with no flow under pressure gradient below a threshold value and Darcian flow under pressure gradient above it. They illustrated a comparison of classical and threshold gradient solutions and showed that the deviation is significant around the corner. Entov et al. [21] developed a numerical method to calculate stagnation zone configuration in different cases.

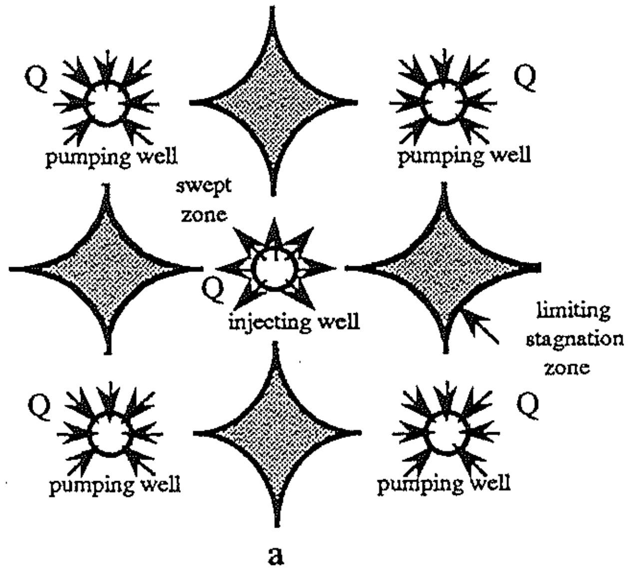


Figure 10a Schematic of limiting stagnation zones for 5–spot well pattern

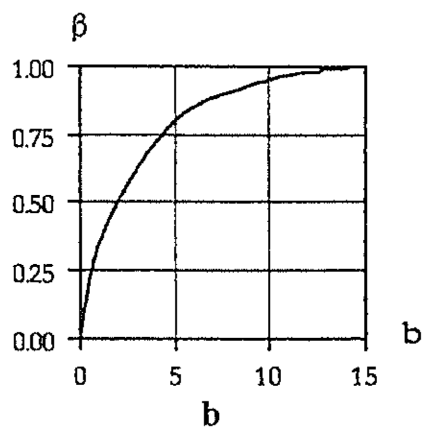


Figure 10b Sweep efficiency $\beta$ vs dimensionless parameter b (from *Bernadiner and Entov*, 1975)



If the porous medium consists of several (say $m$) hydrodynamically unconnected layers of different thickness $d_i$ and permeability $k_i$, the system of equations 6–7 is expanded by the pressure equality condition on the radius $r_w$ of each well, and by a condition stating that the sum of the water flow rates from each layer, $Q_i$, equals the well flow rate $Q$:

$$P_1 = P_2 = \cdots = P_m \qquad \text{at} \quad r = r_w$$
$$\sum_{i=1}^{m} Q_i d_i = QB \tag{8}$$

For porous media with a continuous distribution of permeability across the layer thickness or for hydrodynamically connected layers in the porous medium, average parameters across the medium thickness, such as filtration velocity, permeability, water saturation, and threshold gradient, were introduced. The pressure was assumed to be equal in the vertical at any distance $r$ from the well, thus neglecting vertical flow components (crossflow).

The methods were employed to get analytical solutions for many application problems related to different well patterns. Interested readers can find numerous solutions in the aforementioned publications and in references therein.

## Unsteady Flow

### Equations for Flow in a Confined Reservoir

The first results on unsteady threshold gradient flow under pressure in a confined reservoir were reported by Entov [18]. The filtration law given by eq 3 can be written as

$$(|q| + \lambda)\frac{q}{|q|} = \begin{cases} -\frac{k}{\mu}\nabla p & \text{for} \quad |\nabla p| > G \\ 0 & \text{for} \quad |\nabla p| \le G \end{cases} \tag{9}$$

Equation 9 is substituted in the mass balance equation

$$\frac{\partial(n\rho)}{\partial t} + \operatorname{div}(\rho q) = 0 \tag{10}$$

where $n$ is the porosity and $\rho$ is the fluid density.

Introducing the liquid state equation

$$\frac{d\rho}{\rho} = \frac{1}{E_\rho}\,dp \tag{11}$$

and the equation for the matrix deformation

$$\frac{dn}{n} = \frac{1}{E_n}\,dp \tag{12}$$

the mass balance equation (10) takes the form ($\rho$ is constant in space)

$$\frac{\partial p}{\partial t} + \frac{E}{n}\,\mathrm{div}\;q = 0 \tag{13}$$

where $E = (1/E_n + 1/E_\rho)^{-1}$ is a modulus of bulk compressibility and $q$ is defined from eq 9.

## One-Dimensional Flow

Under constant initial pressure gradient across a one-dimensional porous medium and constant boundary flow velocity, corresponding to injection or inflow of a liquid with threshold gradient $G$, equations 9 and 13 have a self-similar solution yielding

$$p = Gf(\xi);\;\; \xi = \frac{x}{2\sqrt{\chi t}};\;\; \chi = \frac{kE}{n\mu} \tag{14}$$

The normalized pressure distribution versus the distance variable $\xi$ is shown in Figure 11. This solution shows that if the initial pressure gradient is lower than the threshold gradient (as is the case in Figure 10), the pressure disturbances propagate over a finite distance in the porous medium [18, 50]. If the initial pressure gradient is greater than the threshold value, the pressure changes affect a zone of infinite length in the porous medium.

## Radial Flow

Radial non-Newtonian inflow from a reservoir to a single well of radius $r$ pumping at constant rate $Q$ was studied by Entov and Malakhova [19]. They found that the pressure at the well $p_w$ at time $t << t^*$, where

$$t^* = \frac{1}{12\chi}\left[\frac{\mu Q}{2\pi kG}\right]^2 \tag{15}$$



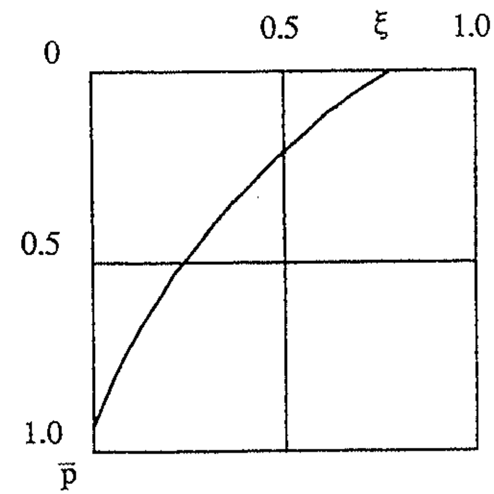


Figure 11 $\bar{p} = \frac{p}{2G\sqrt{\chi t}}$ $\xi = \frac{x}{2\sqrt{\chi t}}$
Normalized pressure distribution in one–dimensional nonsteady flow (from *Bernadiner and Entov*, 1975)

is the same function of time as in the Newtonian flow

$$p_w \approx -\frac{\mu Q}{4\pi k}\ln\frac{12\chi t}{r_w^2} \tag{16}$$

Equation 16 is the well-known Theiss solution for radial flow to a well in a confined reservoir, approximated for large time $t > r_w^2/(0.4\chi n)$ or in terms of a dimensionless time variable $\tau = t\chi/r_w^2$, $\tau > 10$, where a typical porosity value $n = 0.25$ is used. On the contrary, at $t >> t^*$,

$$p_w \approx -G\left[\frac{3\chi t\mu Q}{\pi kG}\right]^{1/3} + \frac{\mu Q}{6\pi k}\ln\frac{\pi Gkr_w^3}{3\mu\chi tQ} + \frac{3\mu Q}{4\pi k} \tag{17}$$

and the pressure changes in time from a logarithmic law to a power law. Estimating the time $t^*$ from graphical display and curve fitting of real data gives the opportunity to estimate the threshold gradient value in reservoir conditions (Figure 12).

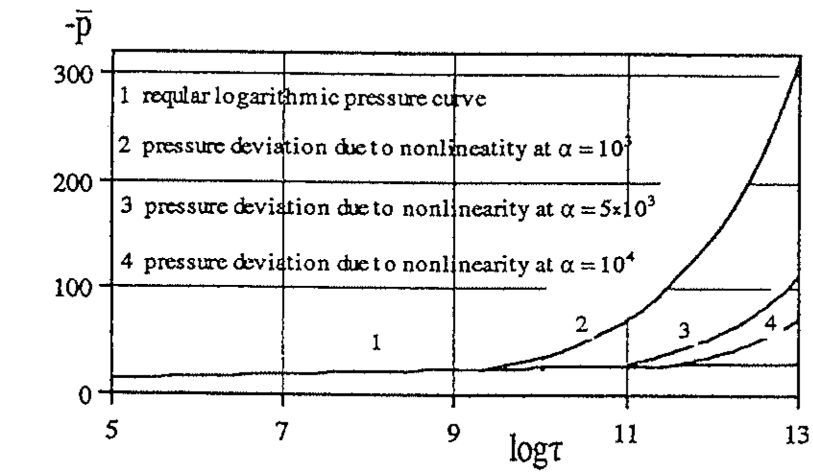


Figure 12 $\overline{p} = \frac{\pi k p_w}{\mu Q}$ $\tau = \frac{\chi t}{r_w^2}$ $\alpha = \frac{Q\mu}{\pi k G B \, r_w}$

Threshold gradient effect on transient pressure in pumping well

## Reservoir Analysis

A practical method to determine the reservoir parameters, namely the unstressed reservoir pressure and the threshold gradient, was developed by Entov et al. [20]. The method consists of analysis of the transient pressure in the well in two phases, pumping and injecting. After pumping stops, the pressure at the well increases until the pressure gradient in the vicinity of the well achieves the threshold value, $G$. At that time the well pressure, $p_{w,1}$, is found from

$$p_{w,1} = p_r - GR \tag{18}$$

where $p_r$ is the (unknown) unstressed reservoir pressure, and $R$ is the radius of zone of influence of the well. During the injection phase the pressure at the well increases further above the unstressed pressure $p_r$. After injection stops the pressure in the reservoir is

$$p_{w,2} = p_r + GR \tag{19}$$



provided that the affected zone around the well is the same for both pumping and injection. The radius, $R$, depends on the injected volume, $V$, because of the reservoir compressibility. To achieve pressure change within the entire zone of influence, the injected volume must be equaled to

$$V = \frac{2\pi R^3 n B G}{3E} \tag{20}$$

From eqs 18 and 19 one may obtain relations for reservoir unstressed pressure and threshold gradient as

$$p_r = \frac{p_{w,2} + p_{w,1}}{2}; \quad G = \frac{p_{w,2} - p_{w,1}}{2R} \tag{21}$$

## Concluding Remarks

It is well established that the Darcy's equation for flow in geologic media, stipulating a linear relation between filtration velocity and hydraulic gradient, is not a universal law, but rather an empirical relation with many exemptions. One type of deviation from Darcy's law occurs when a threshold gradient must be exceeded before flow starts, with the linear relation holding thereafter. This survey has attempted to review the experimental evidence of this behavior of fluid-porous media systems. A non-Darcian dynamic equation is proposed to theoretically model such systems. A variety of results exist today for flow in porous media with threshold gradient, from two-dimensional steady-state situations to one-dimensional and unsteady radial flows, from one-phase to two-phase systems. These results reveal fundamentally different behavior than the well-known description of systems where Darcy's equation is valid. This review aims at promulgating these results to the broader scientific community. It is further recognized that theoretical work has advanced much more rapidly than experimental studies of flows with threshold gradient, particularly at field scale.

After more than thirty years of study of flows with threshold gradient there is still a lack of direct experimental evidence substantiating the existence of stagnation and limiting stagnation zones. This may be why the theory and analytical solutions of the basic equations are often viewed skeptically. An example of this doubt is the section on the threshold gradient literature in the review by Neuzil [39].

In this context, two main directions for future research on the reviewed subject may be proposed. First, it is critical to develop experimental techniques for direct investigation of the moving fluid in a non-Darcian flow regime as well as of the fluid and stagnation zone parameters, especially under field conditions. Second, further development of numerical methods to calculate multidimensional flow with a threshold gradient in homogeneous and heterogeneous geologic media is needed. The primary goal of such studies should be the practical application of numerical models in different engineering problems.

## Notation

| | |
|---|---|
| $q$ | flow velocity vector |
| $k$ | permeability |
| $\mu$ | dynamic viscosity |
| $n$ | porosity |
| $E_n$ | matrix compressibility |
| $E_\rho$ | fluid compressibility |
| $E$ | bulk compressibility |
| $p$ | pressure |
| $G$ | threshold gradient |
| $\omega$ | magnitude of the flow velocity vector |
| $Q$ | pumping or injecting well flow rate |
| $\theta$ | angle between vector velocity q and x axis |
| $\psi$ | stream function |
| $L$ | characteristic distance |
| $B$ | reservoir thickness |
| $R$ | radius of zone of influence of the well |
| $r_w$ | well radius |
| $\rho$ | fluid specific density |
| $\beta$ | sweep efficiency |
| $d_i$ | thickness of the reservoir layer |
| $m$ | number of layers |
| $g$ | specific gravity |
| $\lambda$ | characteristic flow velocity |
| $b$ | well productivity factor |
| $\chi$ | characteristic parameter of the porous medium |
| $V$ | required injection volume for reservoir parameter estimation |



| | |
|---|---|
| $p_{w,1}$ | pressure at well after pumping stops |
| $p_{w,2}$ | pressure at well after injection stops |
| $p_r$ | unstressed reservoir pressure |
| $\nabla$ | gradient of a function |
| $\nabla^2$ | Laplacian operator |
| $\xi$ | self-similar variable |
| $x, y, r$ | coordinates |
| $t$ | time |
| $\tau$ | dimensionless time |

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