# Τυπολόγιο

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$$
\begin{aligned}
&u_{tt} = c^2(u_{xx} + u_{yy}), && 0 < x < l,\ \ 0 < y < m,\ \ t > 0,\\
&u(x,y,0) = f(x,y), && 0 < x < l,\ \ 0 < y < m,\\
&u_t(x,y,0) = 0, && 0 < x < l,\ \ 0 < y < m,\\
&u(x,0,t) = 0, && 0 < x < l,\ \ t > 0,\\
&u(x,m,t) = 0, && 0 < x < l,\ \ t > 0,\\
&u(0,y,t) = 0, && 0 < y < m,\ \ t > 0,\\
&u(l,y,t) = 0, && 0 < y < m,\ \ t > 0.
\end{aligned}
$$

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} K_{nk} \sin\frac{n\pi}{l}x \sin\frac{k\pi}{m}y \cos\left( c\sqrt{\frac{n^2}{l^2} + \frac{k^2}{m^2}}\pi t \right),$$

όπου $$K_{nk} = \frac{4}{lm}\int_0^l \int_0^m f(x,y)\sin\frac{n\pi}{l}x \sin\frac{k\pi}{m} y dxdy.$$

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$x = r\cos\theta$, $\qquad y = r\sin\theta$.

$r = \sqrt{x^2 + y^2}$, $\qquad \theta = \tan^{-1}\dfrac{y}{x}$.

$$\Delta u = \nabla^2 u = u_{xx} + u_{yy} = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}.$$

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$x = r\sin\varphi\cos\theta$, $\qquad y = r\sin\varphi\sin\theta$, $\qquad z = r\cos\varphi$.

$r = \sqrt{x^2 + y^2 + z^2}$, $\qquad \theta = \tan^{-1}\dfrac{y}{x}$, $\qquad \varphi = \cos^{-1}\dfrac{z}{r} = \cos^{-1}\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$,

$$\Delta u = \nabla^2 u = u_{xx} + u_{yy} + u_{zz} = u_{rr} + \frac{2}{r}u_r + \frac{1}{r^2}u_{\varphi\varphi} + \frac{\cot\varphi}{r^2}u_\varphi + \frac{1}{r^2\sin^2\varphi}u_{\theta\theta}.$$

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$$u_t = a^2\left(u_{rr} + \frac{1}{r}u_r\right), \quad 0 \le r < k,\ t > 0,$$
$$u(k,t) = 0, \quad t > 0,$$
$$u(r,0) = f(r), \quad 0 \le r < k.$$

$$u(r,t) = \sum_{n=1}^{\infty} u_n(r,t) = \sum_{n=1}^{\infty} A_n J_0\left(\frac{q_n}{k}r\right)e^{-\frac{q_n^2}{k^2}a^2 t}, \quad \text{όπου} \quad A_m = \frac{\int_0^k r f(r) J_0\left(\frac{q_m}{k}r\right)dr}{\int_0^k r J_0^2\left(\frac{q_m}{k}r\right)dr}, \quad m = 1,2,\ldots.$$

$$x^2 y'' + xy' + (x^2 - p^2)y = 0 \Rightarrow y = AJ_p(x) + BY_p(x)$$

όπου $J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(n+p+1)}\left(\frac{x}{2}\right)^{2n+p},$

$$J_{-p}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(n-p+1)}\left(\frac{x}{2}\right)^{2n-p}, \quad \Gamma(n+p+1) = (n+p)!,$$

$$Y_p(x) = \frac{J_p(x)\cos p\pi - J_{-p}(x)}{\sin p\pi}, \qquad p \in \mathbb{R}.$$

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$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(ru) + \frac{1}{r^2 \sin\varphi}\frac{\partial}{\partial\varphi}\left(\frac{\partial u}{\partial\varphi}\sin\varphi\right) = 0, \qquad 0 \le r < k,\ 0 \le \varphi \le \pi,$$
$$u(k,\varphi) = f(\varphi), \qquad 0 < \varphi < \pi.$$

$$u(r,\varphi) = \sum_{n=0}^{\infty} C_n r^n P_n(\cos\varphi), \quad \text{όπου} \quad C_m = \frac{2m+1}{2k^m}\int_{-1}^{1} f(\cos^{-1}x)P_m(x)dx, \quad m = 0,1,\ldots.$$

$$(1-x^2)y'' - 2xy' + \lambda y = 0 \Rightarrow \quad \lambda = n(n+1), \quad P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n.$$

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$$\Delta\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = r\frac{\partial^2(r\Phi)}{\partial r^2} + \frac{1}{\sin\varphi}\frac{\partial}{\partial\varphi}\left(\sin\varphi\frac{\partial\Phi}{\partial\varphi}\right) + \frac{1}{\sin^2\varphi}\frac{\partial^2\Phi}{\partial\theta^2} = 0.$$

$$\Phi_n(r,\theta,\varphi) = r^n\left(A\cos\mu\theta + B\sin\mu\theta\right)\sin^{\mu}\varphi\frac{d^{\mu}}{dx^{\mu}}P_n(\cos\varphi).$$

$$\frac{d}{dx}\left[(1-x^2)X'\right] + \left[n(n+1) - \frac{\mu^2}{1-x^2}\right]X = 0 \Rightarrow X(x) = P_n^m(x) = (1-x^2)^{\mu/2}\frac{d^{\mu}}{dx^{\mu}}P_n(x).$$

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$$\|ax + \beta y\|^2 = a^2\langle x,x\rangle + 2a\beta\langle x,y\rangle + \beta^2\langle y,y\rangle.$$
$$|\langle x,y\rangle| \le \|x\|\|y\|.$$
$$\|x+y\| \le \|x\| + \|y\|.$$