AN ANALYSIS OF THE U.S. GROSS STATE PRODUCT CO-MOVEMENT USING THE MINIMUM DOMINATING SET

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Keywords: Minimum Dominating Set, Complex Networks, Optimum Currency Areas, Business Cycle Synchronization, Regional Economics

I. INTRODUCTION

In this study we use the Minimum Dominating Set (MDS) in order to identify a subset of states that can represent the initial network of the Gross State Product (GSP) growth rate of 51 US states (50 states and the District of Columbia). This subset can then be used as an auxiliary alert gauge when investigating the macroeconomic effects of GSP movements in inter-state studies or it can be integrated in forecasting models for the US GSP movements, reducing the volume of data needed for such an analysis.

In his work, [2] argues that it may be more efficient for a group of economies to abolish their sovereign currency and adopt a common one. The main advantages that emerge from the sharing of a common currency are a) lower international transaction costs, b) exchange rate risk dissolution and c) increased price transparency. On the other hand, abolishing the sovereign currency induces the inability for a nation to implement independent monetary policy. For the benefits to exceed the costs in a monetary union, it is essential that the business cycles are synchronized. Our empirical analysis provides various findings regarding the business cycle co-movement of the 51 US states.

We contribute to the existing literature in two ways: first we suggest a new tool for monitoring the GSP growth inter-dependence of US states and second we demonstrate the possible utility of applying the MDS approach in complex economic networks.

II. DATA AND METHODOLOGY

Given a graph \( G = (N, E) \), where \( N \) is the set of nodes and \( E \) the set of edges, a dominating set is defined as a subset \( S \subseteq N \) so that every node \( u \in N \) is either included in \( S \) or is adjacent to one or more nodes of \( S \). MDS is called the dominating set with the minimum cardinality.

We collect data for the Gross State Product of 51 US states, in annual rate, for the time period 1997-2010 for which we then calculate the annual growth rates. As a similarity measure we calculate the cross-correlations in two ways: the standard Pearson correlation coefficient \( r_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j} \) and a weighted one, assigning logarithmically heavier weights to more recent GSP values. The weighted correlation coefficient is given by eq. (1) according to [1].

\[
r_{xy} = \frac{\sum_{i=1}^{n} w_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} w_i (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} w_i (y_i - \bar{y})^2}}
\]

where \( x \) and \( y \) are the time-series under consideration and \( w \) is a vector containing the weights.

Afterwards an arbitrary threshold is imposed to the network in order to work only with the high positive correlations. For the purposes of our analysis we consider the nodes with a correlation value below of the given threshold totally uncorrelated and the respective edges are removed from the network. This procedure might produce isolated nodes in the case that a state’s GSP is totally uncorrelated with the rest of the network. In the next step the MDS is identified. Since calculating the MDS is computationally hard we apply the following heuristic method: We consider one binary parameter \( x_i, i = 1, ..., n \) for every node of the network such that \( x_i = 1 \) when node \( i \) is a DS node and \( x_i = 0 \) in the opposite case. A DS is then calculated through:

\[
x_i + \sum_{j \in E(i)} x_j \geq 1, \forall i,
\]

where \( N(i) \) stands for the neighboring nodes set of node \( i \). To consider the DS with the minimum cardinality, we calculate the next equation:

\[
\min_x f(x) = \sum_{i=1}^{n} x_i.
\]

Thus, the calculation of MDS takes the form of finding the binary vector \( x = [x_1, x_2, ..., x_n] \), minimizing (3), under the constrains in (2).

III. EMPIRICAL RESULTS

Table 1 contains the empirical results for the threshold value of \( p = 0.7 \) for the standard and weighted correlation coefficient.

<table>
<thead>
<tr>
<th>TABLE I. SIMULATION RESULTS</th>
<th>Standard</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network Edges</td>
<td>401</td>
<td>497</td>
</tr>
<tr>
<td>MDS size</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Isolated Nodes</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Multiple conclusions arise from our analysis: First we observe that the systemic behavior of the complete 51 state network can be represented by a smaller group of 7 or 5 states in the cases of standard and weighted correlations respectively. Second, in both correlation approaches there exists a group of 3 isolated nodes indicating 3 totally uncorrelated states (namely Delaware, Louisiana and North Dakota), in terms of GSP growth rate co-movement. For the implementation of an efficient monetary policy these rates need to be synchronized. Therefore, apart from the need for a higher degree of synchronization within the remaining network, special attention should be given to these isolated nodes. Finally, we observe that in the case of the weighted coefficient, the number of edges that survive (the same in both cases) imposed threshold, increases. This might be interpreted as evidence of macroeconomic convergence between US states as it indicates possible higher GSP growth correlations in recent years.

REFERENCES