# Large Scale Cyclonic Vortices of the Submerged Gravity Intruding Sewage Field from a Large Diffuser in a Stratified Sea 

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#### Abstract

Summary Large scale submerged intruding currents are studied where the Rosby number is small enough, so that the Coriolis forces are important. Example of such flow in the ocean is the buoyant flow from a large sewage diffuser in a stratified sea. For simplicity we examine the outflow from a round outfall. The buoyant jet flow rises until it reaches an equilibrium depth at which the convective flow began to propagate radially outward horizontally at its neutral level, forming an intruding gravity current (or patch or slug), which initially is almost axisymmetric, but later on, with increasing the radius of the patch, the Coriolis force increases, and consequently large cyclonic vortices are formed. The submerged sewage field from large diffusers (for example Los Angeles, Athens, Boston) attains large length scales and the Coriolis force is certainly important in the study of the transport of the pollutant residuals. This flow is important from the environmental point of view because the intruding gravity current transports the effluent's suspended sediment and pollutants that may coagulate and deposit around the outfall at the sea floor. We present some preliminary results of laboratory experiments in a large rotating table to study the spreading rates of the intrusive gravity currents produced by a constant inflow into a stratified body of water rotating as a solid body. The experiments were conducted over a wide range of initial parameters (initial kinematic mass flux, initial kinematic buoyancy flux, rotation period, stratification). In this paper we present experimental results about various parameters which characterize the flow morphology, i.e. the increase of the typical length and total area of the intrusion as a function of the time, and photos to observe the evolution of the spreading and when the geometry of spreading changes from axisymmetric patch to ellipsoid and subsequently to an ellipsoid, which generates large cyclonic vortices.


## Keywords:

Large sewage outfall, cyclonic vortices, coriolis force, gravity current

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## Introduction - Theoretical Considerations

In this paper we study experimentally the axisymmetric intrusion and flow pattern of a gravity current generated by the continuous release of buoyant fluid at the bottom of a stratified rotating tank. This flow simulates the outflow from large diffusers in a stratified sea environment of effluents from a waster treatment plant. The flow from a sewage outfall in a stratified sea rises as high as its momentum and buoyancy will carry it, and then it spreads horizontally at its neutral level, forming an intruding gravity current (or lens or patch or slug). The basic objective of this paper is the laboratory study of the spreading of three dimensional submerged gravity current that is produced by the continuous source of fluid in a stratified large tank rotating anticlockwise with angular velocity $\Omega$. The water in the tank is density stratified and it rotates as solid. For given fluid properties, this problem has three adjustable parameters: the Coriolis parameter f , the flow rate Q and the buoyancy frequency N .

The growth of the spreading gravity current results from the balance of the following four forces: the pressure (buoyancy) force $F_{p}$, the inertia force $F_{i}$, the Coriolis force $F_{c}$ and the interfacial friction force $F_{d}$. The force, which drives the flow is only the pressure (or buoyancy) force $F_{p}$. The forces which retard (or resist) the flow are three: the inertia of the gravity current $\mathrm{F}_{\mathrm{i}}$, the Coriolis force $F_{c}$ and the friction force $F_{d}$. Laboratory observations indicate that the spreading for "small" times grows as an axisymmetric radial gravity current, resembling the intrusion in the absence of rotation (see Kotsovinos [1]). For larger times the plan view of the intrusion resembles an ellipsoid, with axes, which increase with time. At even larger times two symmetrical cyclones are formed at the two ends of the large axis of the ellipsoid.

By integrating the vertical component of the momentum equation over the spreading patch (slug) and by neglecting small terms (i.e. change of vertical inertia) we obtain the physically expected result that the total weight of the slug balances the total pressure force, which acts on the slug surface S , i.e.
$\iiint_{V} \rho_{s}(\vec{x}) g \vec{k} d V=\iint_{S} p(\vec{x}) \vec{n}(\vec{x}) d S$
where $\rho_{s}(\vec{x})$ is the density at any point within the slug, $p(\vec{x})$ is the ambient pressure at any point $\vec{x}$ at the interface of the slug, $\vec{n}(\vec{x})$ is the unit vector perpendicular to the surface, g the gravitational constant and $\vec{k}$ is the unit vector in the vertical direction. Since in a rotating frame, the ambient pressure depends on the ambient density profile, the depth, the axial distance and the rotation angular velocity $\Omega$, it is clear that equation (1) imposes a relationship between the density within the slug and the ambient density.
Since the density within the slug is not known, it is assumed that to the first approximation the density within the slug varies linearly with the depth and that the ambient density varies also linearly with the depth. Subsequently we integrate equation (1) in a slug of constant depth H and radius R and we find the following relationship between the ambient and slug densities:

$$
\begin{equation*}
\rho_{\mathrm{au}}+\rho_{\mathrm{al}}=\rho_{\mathrm{u}}+\rho_{\mathrm{l}} \tag{2}
\end{equation*}
$$

Where $\rho_{\text {au }}$ and $\rho_{\mathrm{al}}$ are respectively the densities of the ambient fluid at the upper and lower interfacial layer of slug, and $\rho_{u}$ and $\rho_{l}$ are respectively the densities at the upper and lower interfacial layer within the slug. Similar equations to equation (2) can be found for various combinations of ambient and slug density profiles. For example assuming that the density of the fluid in the spreading slug is constant and equal to $\rho_{\mathrm{s}}$ and assuming again linear ambient stratification, we find that
$\rho_{\mathrm{s}}=\rho_{\mathrm{u}}=\rho_{\mathrm{l}}=\left(\rho_{\mathrm{al}}+\rho_{\mathrm{au}}\right) / 2$

Equation (2) can be written as

$$
\begin{equation*}
\rho_{\mathrm{u}}-\rho_{\mathrm{au}}=\rho_{\mathrm{al}}-\rho_{\mathrm{l}}>0 \tag{4}
\end{equation*}
$$

which implies that the density within the slug at its upper part is larger than the ambient density at the same level. The opposite is observed in the lower part of the slug. The measurements taken in this investigation (see subsequently Figures 2a to 2d) indicate that the density profile within the slug is to the first approximation linear, and also the validity of equation (4).

Simpson [2] reviewed gravity currents, which occur whenever fluid of one density flows primarily horizontally into fluid of a different density, in a non rotating ambient fluid. The case that we study has not been studied sufficiently in the past. The vertical buoyant jet rises and it stops at its neutral density level, and subsequently an intruding gravity current is produced driven by the density difference between the intruding fluid and the surrounding ambient. The action of Coriolis forces in this problem is very important in the evolution of these submerged intruding gravity-driven flows.

When gravity current, (produced by the continuous influx of buoyant source and driven basically by density differences) evolves in a rotating environment, the nature of its motion is very different from that which arises when the flow takes place in a non rotating environment. In the presence of rotation, and when the Rosby number is small, the Coriolis effects deflect the flow and the intruding fluid is subject to centrifugal accelerations, which inhibit radial spreading. Therefore the slug tends to increase in depth and to form an axisymmetric lens of dense fluid, which at some point becomes unstable, takes the form of an ellipsoid, and develops discrete eddies on the length scale of the Rosby radius. Hogga et al [3] studied the axisymmetric propagation of a relatively dense gravity current of a given initial volume over a horizontal rigid boundary when the intruding fluid is a suspension of heavy particles and the ambient fluid is steadily rotating about a vertical axis.
Narimousa [4] has studied the turbulent convection into a two-layer stratified ambient fluid where penetrative convective flow interacted with a density interface. It has been found that the Richardson number of the system is a major factor for predicting the condition under which the convective flow penetrates the density interface. Narimousa [5] has studied the penetrative turbulent convection from a localized circular top source into a rotating, linearly stratified ambient fluid in a laboratory tank. Initially, the induced three-dimensional convective flow penetrated rapidly into the stratified water column until it reached an
equilibrium depth at which the convective flow began to propagate radially outward. At this stage, the usual cyclonic vortices were generated around the convection source at the edge of the radially propagating flow. Soon after, a thin subsurface anticyclone was formed at the level of equilibrium depth beneath the convection source.

## 1. Experimental Setup

The laboratory experiments were conducted in a large cylindrical tank 520 cm in diameter and 100 cm deep, which was mounted on a turntable. First the lower layer was formed, i.e. a layer of salty water was fed into the tank. The thickness of the lower layer was 20 cm . The lower layer was left for one hour to reduce ambient water movements. The upper layer was then prepared. Tap, freshwater water was slowly (for about 5 hours) poured at the free surface of the salty layer, so that to form a top lighter layer of 10 cm thickness. A small ( 4 mm ) orifice at the centre of the base of the tank was connected via a feed line to a constant-head reservoir of tap water dyed with potassium permanganate, which would form the buoyant jet during the experiment. The dyed water, which was to form the buoyant jet was then prepared in a constant head reservoir which was mounted on the rotating table.

The system is brought to a solid-body rotation (counter-clockwise looking from above). The rotation period could vary from 25 to 120 sec . It usually took about 12 h to bring an experimental setup into a solid-body rotation and to allow movements within it to reduce to the minimum possible in a laboratory setting (typically 0 ). When solid body rotation rate was achieved, the initial temperature and salinity profiles within the stratified tank (density distribution) were measured using a micro conductivity probe (micro scale conductivity and temperature probe, manufactured by Precision Measurement Engineering, USA, Model 125, temperature resolution $10^{-3}{ }^{\circ} \mathrm{C}$, conductivity resolution $2 \times 10^{-5} \mathrm{Sm}^{-1}$, time constant respectively 0,02 and $0,004 \mathrm{~s}$ ), mounted on the rotating table and remotely controlled. The conductivity temperature probe was remotely operated and was traveling vertically downward and upward, and the data were stored in a PC, to calculate the density of the water column, at various places and times during the experiment. Subsequently, the valve in the line feeding the plume fluid into the tank was opened and the plume began to emerge vertically. The volume flux of the buoyant jet was measured using a magnetic flow meter. The experiment was continued in every case until the intruding gravity current or the vortices reached close to the end of the tank.

Two CCD video camera were mounted on the rotating table and continuously recorded the experiment, so that their field of view would capture the initial growth and spread of the intruding submerged gravity current. A grid was drawn at the bottom of the tank so that the video data could be calibrated.

## 2. Experimental Results

The buoyant jet initially rose through the lower layer about a vertical centre line, demonstrating that for these particular experiments (high Froude number, small thickness of the lower layer and slow rotation about an axis parallel with the jet flow) the rotation motion and the Coriolis forces do not play any significant role on the evolution of the vertical buoyant jet. The buoyant jet spreads into a cone-shaped configuration through lateral
turbulent mixing with the ambient water. The buoyant jet flow rises until it reaches an equilibrium depth at which the convective flow began to propagate radially outward horizontally at its neutral level, forming an intruding gravity current. The rotation of the earth alters the path and structure of the gravity current. The dynamics of this flow is influenced by the earth rotation due to the scale of the problem. In the absence of rotation the intruding fluid spreads radially in a thin layer (see Kotsovinos [1]). The submerged gravity current is initially approximately axisymmetric. It is interesting to note that initially the flow morphology resembles the flow morphology of the non rotating intrusion. However, later on the flow morphology takes progressively the form of an ellipsoid, which later on develops baroclinic instabilities at its fronts, and eventually at the edge of the front two large, almost symmetric cyclonic vortices are formed (see Figure 1). In our experiments the cyclonic vortices are wellorganized, and eventually propagate away from the gravity current (patch) allowing a new generation of cyclonic vortices to form at the front of the patch. This process will continue to repeat itself for as long as the convection source is active. Baroclinic instabilities have also observed by previous investigators, Coates et al. [6]; Visbeck et al. [7]; Whitehead et al. [8], Narimousa [4], Narimousa [9] and Narimousa [5].

The density of the water column at a certain location (penetrating the gravity current) during the running of an experiment is shown in Figures 2a to 2d. In each Figure the blue line is the density distribution 2 min before the emergence. The second line is the density distribution within the slug at dimensionless times $\mathrm{t} / \mathrm{T}=4,6.4,9.6$ and 16.8 revolutions. Comparing these figures, it is interesting to observe the growth of the thickness of the slug. The flow morphology at the time of the measurements was an ellipsoid. We see linear ambient density stratification, while the density within the slug at its upper part is larger than the ambient density at the same level in all figures. The opposite is observed in the lower part of the slug. The measurements taken in this investigation indicate that the density profile within the slug is to the first approximation linear, and also the validity of equation (4).

In Figure 3 we present the growth of the surface of the slug as a function of the dimensionless time. In Figure 4 the growth of the two axes of the slug as a function of the dimensionless time are shown. We observe that for the first seven rotation periods the two axes are almost equal, i.e. to the first approximation the gravity current spreads axisymmetrically. Later on, the gravity currents spreads faster in one direction and the flow morphology changes to an ellipsoid.


Figure 1. Successive photo of the evolution of the submerged gravity current. The rotation period is $\mathbf{6 0} \mathrm{s}$. The first photo was taken 30 s from the start of the buoyant plume flow. The time distance between each photo is half rotation period ( 30 sec ). We observe that initially the spreading of the gravity current is axisymmetric, and that at about 3 revolutions the gravity current has the morphology of an ellipsoid. At about five revolutions, we observe the initial formation of two cyclonic vortices. At about six revolutions, the cyclonic vortices are well developed. At about 8 revolutions, the well-organized cyclonic vortex has moved away from the main gravity current and a process is under way for the production of two new cyclonic vortices.


Figure 2a. Density distribution: a) 2 min before the emergence of the buoyant jet (blue line) and b) $5 \mathbf{m i n}$ later within the slug (t/T=4 revolutions ).


Figure 2b. Density distribution: a) 2 min before the emergence of the buoyant jet (blue line) and b) $8 \mathbf{m i n}$ later within the slug ( $\mathbf{t} / \mathrm{T}=6.4$ revolutions).


Figure 2c. Density distribution: a) 2 min before the emergence of the buoyant jet (blue line) and b) 12 min later within the slug ( $\mathbf{t} / \mathrm{T}=9.6$ revolutions).


Figure 2d. Density distribution: a) $2 \mathbf{m i n}$ before the emergence of the buoyant jet (blue line) and b) 21 $\mathbf{m i n}$ later within the slug ( $\mathbf{t} / \mathrm{T}=16.8$ revolutions). Comparing this Figure with the previous Figures 2a to 2c, it is interesting to observe the growth of the thickness of the slug. The flow morphology at the time of the measurements was an ellipsoid. The two cyclones were not developed.


Figure 3. The growth of the surface of the slug as a function of the dimensionless time.

surf30 b with t T

Figure 4. The growth of the two axes of the slug as a function of the dimensionless time. We observe that for the first seven rotation periods the two axes are almost equal, i.e. to the first approximation the gravity current spreads axisymmetrically. Later on, the gravity currents spreads faster in one direction and the flow morphology changes to an ellipsoid.

## Conclusions

The macroscopic preliminary experimental observations of the trajectory and the contours of the submerged gravity current in a rotating tank reveal the appearance of various regimes and lateral instabilities. The submerged gravity current is initially approximately axisymmetric. It is interesting to note that initially the flow morphology resembles the flow morphology of the non rotating intrusion. However, later on the flow morphology takes progressively the form of an ellipsoid, which later on develops well-organized baroclinic instabilities at its fronts, which eventually propagate away from the gravity current.

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