GRAVITY CURRENTS IN A DENSITY STRATIFIED POROUS AQUIFER

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ABSTRACT

This paper presents the results of laboratory experiments conducted to study the two dimensional intrusive gravity currents produced by a constant inflow into a density stratified aquifer. A semi-empirical solution is offered for the flow produced in density stratified aquifer due to the artificial recharge by a two dimensional source of constant strength (line source), embedded in the porous medium of uniform porosity. The artificial groundwater recharge scheme examined in this study, is accomplished by injection from a perforated pipe in a linearly density stratified porous medium of uniform porosity. The density of the recharging water is assumed equal to the density of the stratified ambient fluid at the level of the recharging pipe source. It is observed that the intrusion forms a thin layer at the elevation of the source. Balances of the forces that drive and retard the flow indicate that the intrusion is characterized only by one spreading regime: the pressure (buoyancy) force balanced by the viscous Stokes forces on the numerous grains. It is found theoretically that the length of the intrusion L (t) increases with time according to the relation $L(t) \sim t^m$, where $m=2/3$. The experimental results seem to confirm the derived spreading relation.

KEYWORDS
Porous media, artificial recharge, density stratified aquifer, intrusion, gravity current.

1 INTRODUCTION

Artificial recharge (i.e. the artificial increasing of the amount of surface water entering an aquifer) is used for many purposes. The most important of these are
a) The improvement (purification) of water quality, b) storage of excess water from wet periods for subsequent use in dry ones, c) maintenance of ground water levels and d) disposal of treated unwanted water.

Most of published knowledge of the state of the art of the hydrodynamics of artificial recharge assumes that the water of the aquifer has a uniform density or assumes sharp fresh –salt-water ambient water; see for example Mahesha (1998), Thompson et al (1999). . Little attention has been given to the case of the artificial recharge of an aquifer where the density of the ground water is linearly stratified. This situation may appear in a coastal region.

The basic objective of this paper is the stratified aquifer hydrodynamics of the artificial recharge by a two dimensional source of strength 2Q units of volume per unit time per unit length (line source), embedded in a porous medium, in a linearly density stratified porous medium (aquifer) .It is assumed that the porous medium has a constant intrinsic permeability k and is saturated with water of viscosity μ. It is assumed that the artificial recharge scheme examined in this study, is accomplished by injection from a perforated long pipe in a linearly density stratified porous medium of uniform porosity. The density of the recharging water is assumed equal to ρo, which for simplicity is assumed equal to the density of the ambient fluid at the level of the recharging pipe source. The above configuration resembles the artificial groundwater recharge by injection using radial collector wells,proposed by Huisman et al (1983) .

In this paper we present relationships for the growth of the longitudinal length L(t) of the gravity current, intruding the saturated aquifer ,as a function of time. After the entry phase, the recharged water will flow more or less horizontally through the aquifer over extended distances, involving long periods of time. The behavior of submerged intrusions due to artificial recharge is of interest to hydraulic and environmental engineer, because it is related with the underground water quality.

2. AN ORDER OF MAGNITUDE ANALYSIS OF THE GROWTH L (t) OF THE LENGTH OF THE GRAVITY CURRENT

2.1 Continuity equation

The groundwater is stagnant and linearly stratified, so that the density, ρ (z), decreases with increasing elevation z .The porous medium is assumed homogeneous. Two dimensional Cartesian co-ordinate axes are chosen with the z-axis directed vertically upwards. we may assume that the line source of water volume flux is a perforated pipe; the recharge is achieved by applying pressure to the water in the perforated pipe. The volume flux per unit length of the pipe is kept constant and equal to 2Q, it starts at the time t=o and the density of the recharged water is equal to ρo, where ρo is the density of the stratified fluid at the level of the pipe location.

The flow out of the pipe in the porous medium spreads horizontally at its neutral level and forms an intruding submerged gravity current in the saturated, density stratified porous medium.

It is assumed that the flow out of the perforated pipe at x=0, impinges the ambient porous media and the ambient stratified water, rises and descends, and it produces the initial region of the intruding patch.

It is reasonable to distinguish two regions: (1) the impingement region where the flow out of the pipe establishes the initial condition for the intruding water; (2) the main spreading region, which is outside the impingement region.

It may be argued that entrainment in the impingement region is small, and therefore that approximately the volume of the slug (gravity current) at time t is given by the following equation:

$$\text{Volume of slug} = 2Qt \qquad (1)$$

If it is assumed that the typical vertical and horizontal extent of the two dimensional intruding fluid are H and 2L respectively, then (1) gives

$$HL=Qt \qquad (2)$$

2.2 Vertical momentum equation

By integrating the vertical component of the momentum equation over the spreading patch (slug) and by neglecting small terms (i.e. change of vertical inertia) we obtain the physically expected result that the total weight of the slug balances the total pressure force, which acts on the slug surface S, i.e.

$$\iiint_V \rho_s(\vec{x})\ g\,\vec{k}\,dV = \iint_S p(\vec{x})\,\vec{n}(\vec{x})\,dS \qquad (3)$$

Where $\rho_s(\vec{x})$ is the density at any point $\vec{x}$ within the slug and $p(\vec{x})$ is the hydrostatic ambient pressure at any point $\vec{x}$ at the interface of the slug; $\vec{n}(\vec{x})$ is the unit vector perpendicular to the surface, g the gravitational constant and $\vec{k}$ is the unit vector in the vertical direction. Since the hydrostatic ambient pressure depends on the ambient density profile (and the depth), it is clear that equation (3) imposes a relationship between the density of the slug and the ambient density.

It is assumed that to the first approximation the density within the slug varies linearly with the depth, so that it is easy to integrate equation (3) in a slug of constant depth H and length L(t) to find the following relationship between the ambient and slug densities:

$$\rho_{au} + \rho_{al} = \rho_u + \rho_l \tag{4}$$

where ρ_{au} and ρ_{al} are respectively the densities of the ambient fluid at the upper and lower interfacial layer of slug, and ρ_u and ρ_l are respectively the densities at the upper and lower interfacial layer within the slug.

Since the intrusion layer is neutrally buoyant at the height where it spreads horizontally it is, strictly speaking, the "squeezing" pressure p_u , p_l exerted on the upper and lower surfaces of the slug which oblige the slug to spread.

For linear ambient stratification and linear density profile within the slug, i.e. assuming that the density within the slug of thickness H is given by

$$\rho_u + \frac{\rho_l - \rho_u}{H} z \tag{5}$$

where the vertical distance z measures depth from the upper point of the slug, we find that the pressure inside the slug is equal to

$$p_u + \rho_u g z + \frac{(\rho_l - \rho_u) g}{2H} z^2 \tag{6}$$

The pressure inside the slug is clearly greater than the pressure outside, and therefore the excess horizontal pressure (usually called "buoyancy ") force F_p per unit width which drives the spreading (intrusion) is given by

$$F_p = \rho' g H^2 \tag{7}$$

where $\rho' = (\rho_u - \rho_{au})/6$ (8)

Using the continuity equation (2), equation 7 becomes

$$F_p = \rho' g Q^2 R^{-3} t^2 \tag{9}$$

2.3 Horizontal momentum equation-scaling analysis

The methodology that we will follow to find the asymptotic growth rate of the length L(t) with time is based on the balance of the forces, which drive and retard the flow. Similar methodology has been used previously by Hoult (1972) , Chen and List (1976) Didden and Maxworthy (1982), Lemkert and Imberger (1993) .

The force, which drives the flow, is only one: the pressure (or buoyancy) force Fp. The force, which retards (or resist) the flow is only one, the drag F_{drag} which is exerted by the ambient porous media and the ambient fluid on the intruding fluid (clearly the inertia of the intruding gravity current is negligible).

. Subsequently we find the scaling of the above-mentioned forces, where the continuity equation (2) has been considered and where L(t)/t gives the typical horizontal velocity U within the intrusion, where t is the time.

We find for the pressure force :

F_p = pressure (or buoyancy) force = $O(\rho' g H^2) = O(\rho' g Q^2 L^{-2} t^2)$ (10)

We assume that the drag force, which is applied to the intruding slug, is due to Stokes drag forces of the slug fluid due the flow around the numerous grains of the porous media. The Stokes force F_s due to the laminar flow around a sphere for small Reynolds number (Re<1) , is given by

$F_s=3\pi d\mu U$

where μ is the dynamic viscosity , U the fluid velocity and d is the grain diameter .

We obtain therefore

F_{drag} = drag force $=O(\mu\, d\, U\, n) =O(\mu\, d^{-2}\, LT^{-1}\, HL)=O(\mu\, d^{-2}\, L\, Q)$ (11)

where n is equal to the number of grains within the volume of the slug , i.e. $n\approx HL(t)/d^3$.

We consider below the growth L(t) under the balance of the corresponding driving and resisting force. Clearly we have a balance of the buoyancy driving force F_p and the resisting drag force F_{drag} , and we obtain:

F_p = pressure (or buoyancy) force = F_{drag} = drag force , or

$O(\rho' g Q^2 L^{-2} t^2) = O(\mu\, d^{-2}\, L\, Q)$, which gives

$$L(t) = c_0(\rho' g Q d^2/\mu)^{1/3} t^{2/3} \quad (12)$$

where c_0 is an experimental parameter .

The above analysis predicts the typical horizontal length L(t) of the gravity current increases with time as $t^{2/3}$.Subsequently we describe experiments conducted to test the above asymptotic law.

3. EXPERIMENTAL PROCEDURE

A Hele-Shaw cell was constructed of two sheets of 1 cm optically flat plate glass approximately 66 cm long and 48 cm height , clamped 0.1 cm apart .The entire cell was embedded in a tank 20 cm wide and 100 cm long. A vertical wall, made of glass , 19.9 cm wide and 48 cm height, as shown in Figure 1 , separated the tank in two smaller tanks, tank 1 and tank 2. The left end of the cell was freely exposed in tank 1 , and the right end of the cell was sealed with plastic tape except of a small hole at mid elevation. The hole is temporarily blocked, and it is unblocked at the start of the experiment. Tap water and commercial salt was used to stratify the tank. The tank and the cell was linearly stratified by running layers of successively increasing density in the bottom and then allowing the tank to stand for about 12 hours to smooth out the discontinuities. The actual stratification in the tank was measured with a calibrated conductivity probe.

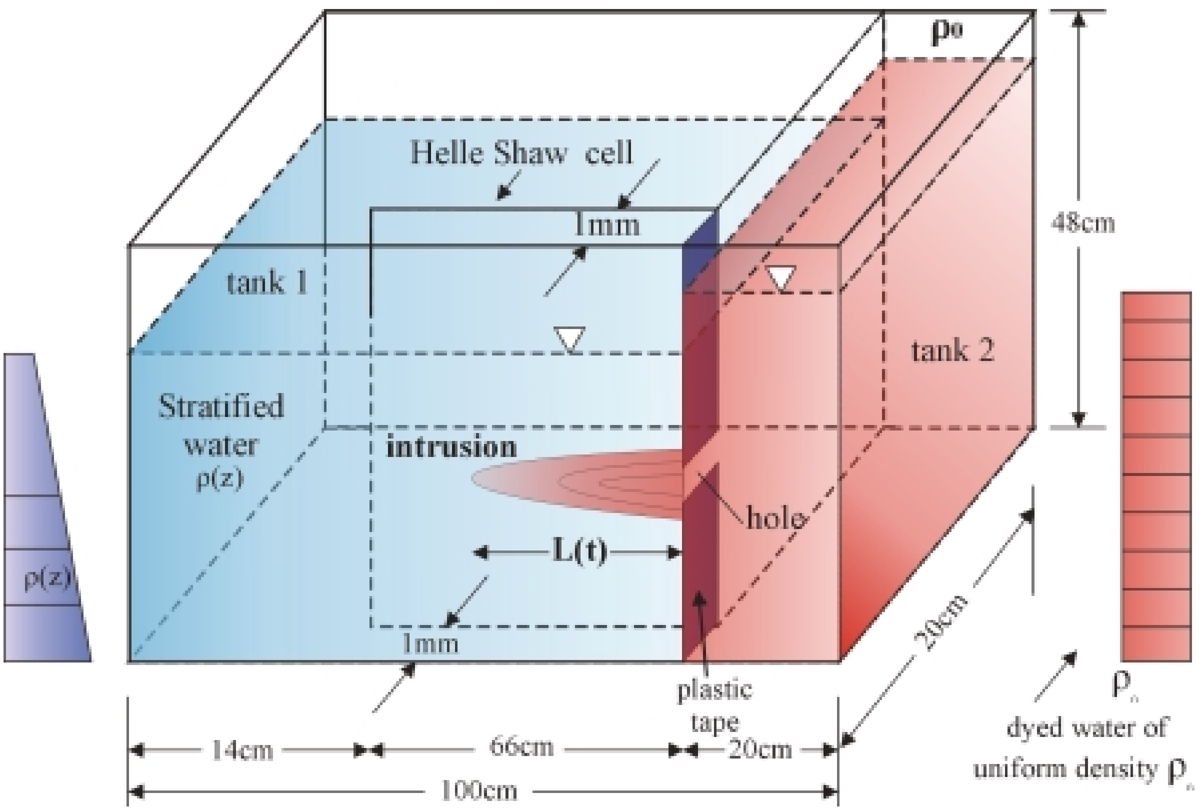

Figure 1 . Experimental configuration (not to scale) of the simulation of the intrusion in a density stratified porous medium.

A typical profile of the density distribution is shown in Figure 2. The density in the tank 2 was uniform and equal to the density at the middle point of tank 1.A red dye was also diluted in the tank 2. The free surface elevation of tank 2 was 1 to 2 cm higher than the free surface elevation of tank 1 . This difference of the free elevation of the two tanks drove the intrusion into the Hele- Shaw cell . A measure of the gradient of the ambient density profile at the level of spreading is given by the Brunt-Vaissala frequency N, which is calculated by the relation

$$N = [\ \frac{g\, d\rho(z))}{\rho(z)}\ ]^{1/2} \qquad (13)$$

When a uniform density gradient had been established in tank 1, the hole which separates the linearly stratified Hele-Shaw cell from the tank 2 , was unblocked and the dyed water from tank 2 started to intrude between the plates of the Hele-Shaw cell. In some experiments the water in the tank 2 was not dyed and the intruding flow was visualized by dropping dye particles between the plates of Hele-Shaw cell.

4 .EXPERIMENTAL RESULTS

For each experimental run , the length L(t) of the intrusion patch was measured as a function of time . A typical contour of the intruding gravity current as a function of time is shown in Figures 3. For each experiment the length L(t) of the spreading slug was plotted in logarithmic scale as a function of time. The raw data of the length L(t) as a function of time t from many experiments are plotted in Figures 4 and 5 .Best-fit lines are also drawn and the corresponding equations of the fits are printed in the plot.

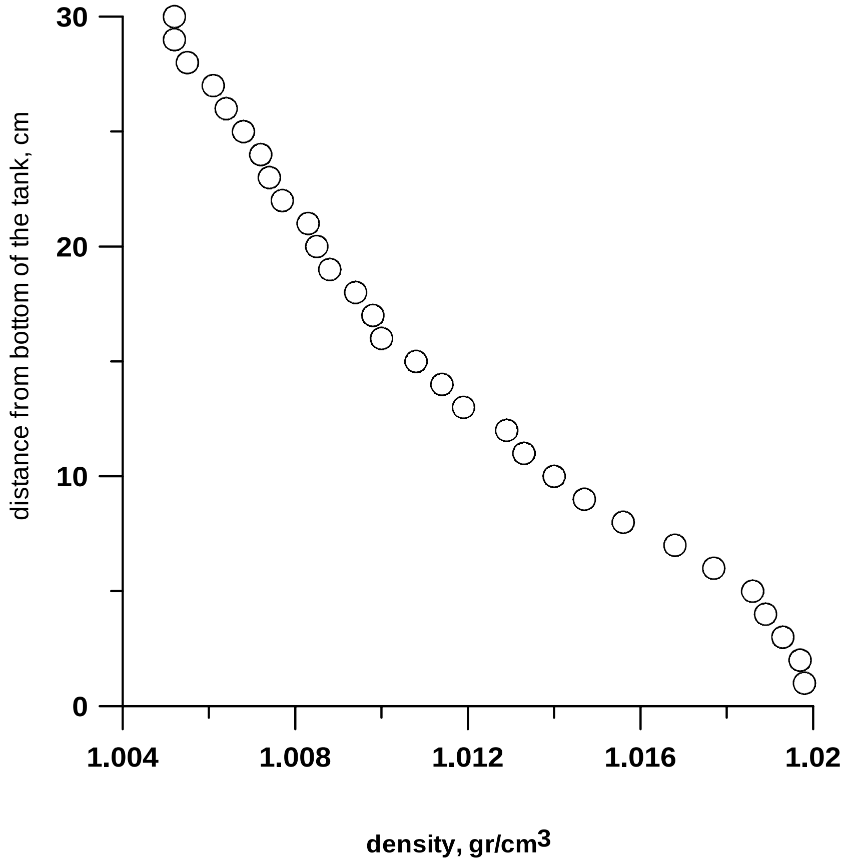

Figure 2. Density distribution in the Helle-Shaw cell. Run B13.
Brunt -Vaissala frequency $N=0,71s^{-1}$.

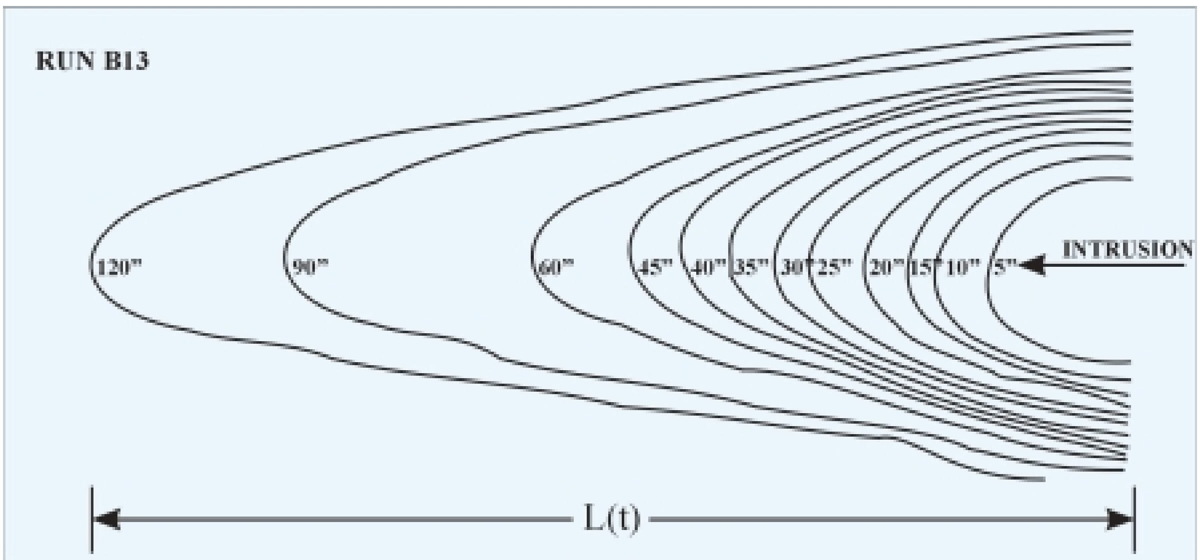

Figure 3 . Contours of the intrusion as a function of time; Run B13; $N=0.71 \text{ sec}^{-1}$.

The best fit equation is of the form $L \sim t^m$,where m is the exponent of time . It can be seen (see Figure 4 and 5) that experimental results show that the length L(t) grows like t^m , where m=2/3, in agreement with our theoretical prediction. In addition , the area within each contour was measured .It was found that the area increased linearly with time, so that we verified that the input volume flux Q was kept constant during the experimental run (equation 1).

5. CONCLUSIONS

The flow visualization indicated that the intruding gravity current occupies a thin layer at the elevation of the hole . This implies that in a practical situation , contaminants that may be present in the artificially recharged water can travel very long distances , practically without any further dilution.

The experimental results indicate that the length L(t) of the intrusion increases with time as $t^{2/3}$, in agreement with the theoretical prediction.

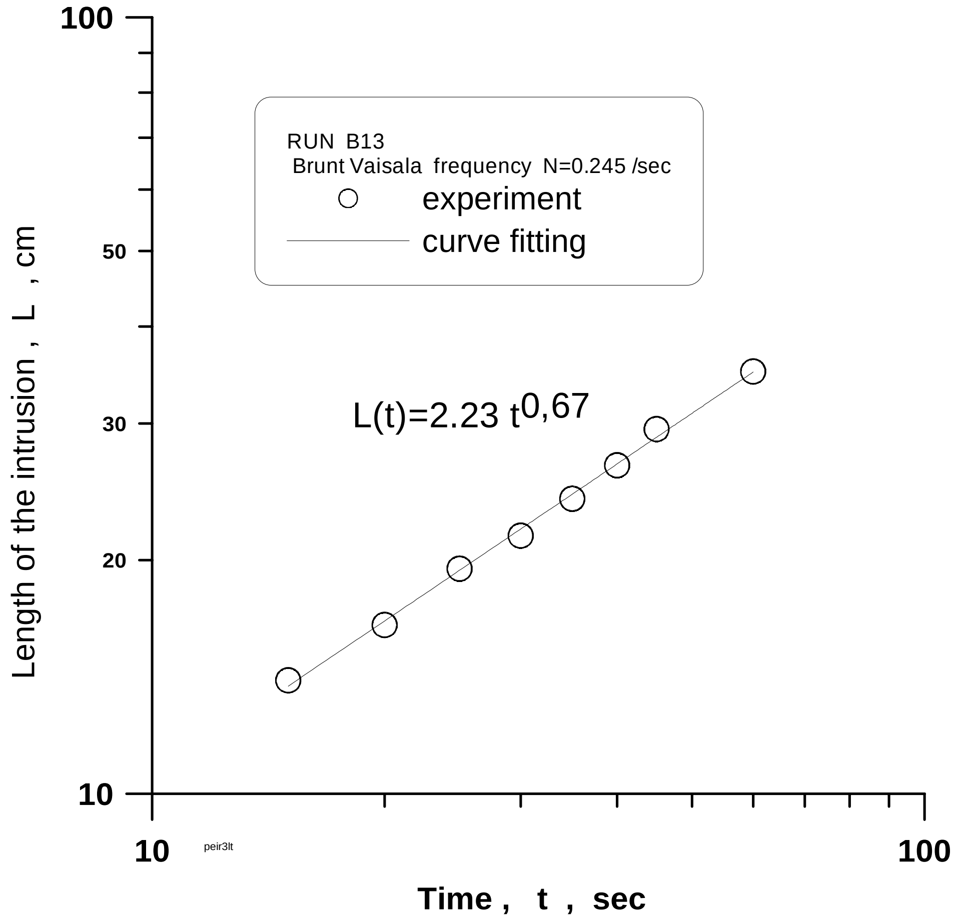

Figure 4 .Length of the intruding gravity current as a function of time .

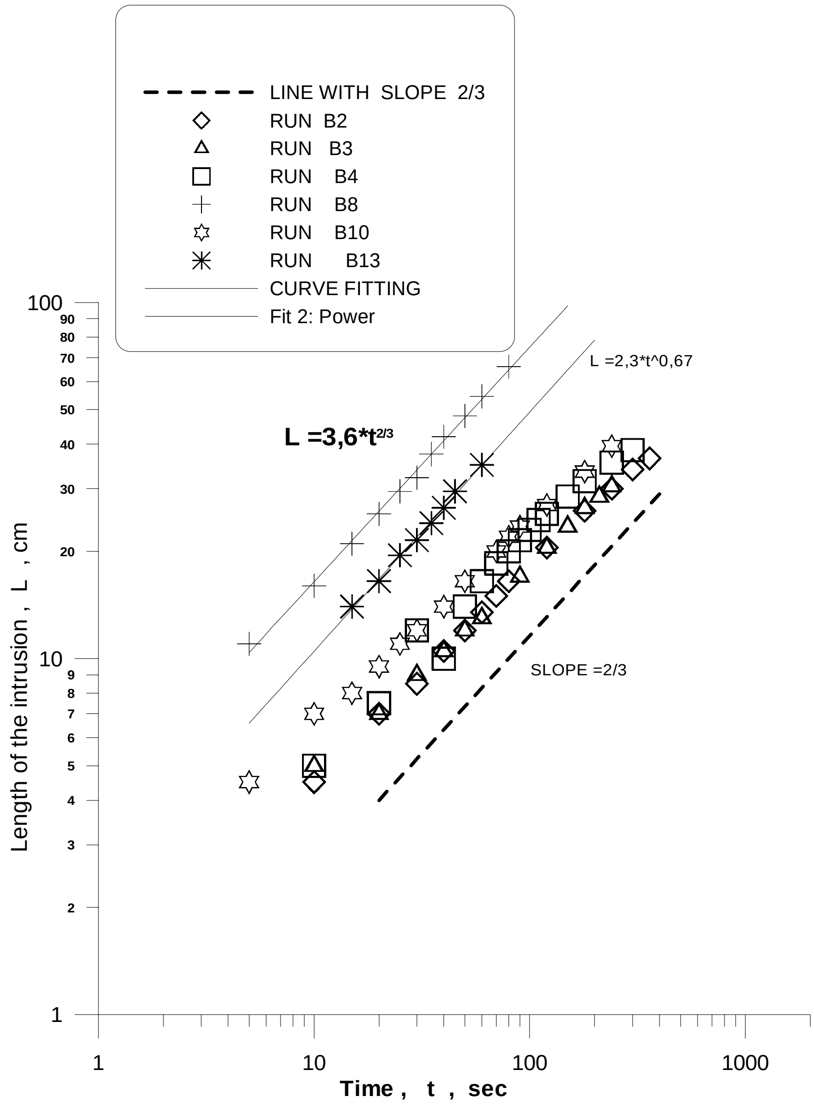

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Figure 5 . Growth history of the the increase of the length of the intrusion with time . Comparison of the experimental results with the slope 2/3 of the theoretical prediction.

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