Paper Title: PHASE RECOVERY OF A STOCHASTIC POINT PROCESS SYSTEM

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PHASE RECOVERY OF A STOCHASTIC POINT PROCESS SYSTEM

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Abstract: *In this work we present two algorithms for the estimation of the phase of a neuroelectric system of point processes using the third-order spectral density function of the output. The neuroelectric system, which is called muscle spindle, plays an important role in the initiation of the movement and the maintenance of the posture. The system can be modelled with the help of a linear stochastic model. The phase of the transfer function is estimated with the help of the third-order spectral density function of the output. The estimate of the third-order spectral density function is obtained by smoothing the third-order modified periodogram statistic. As illustrative examples we examine the behavior of the muscle spindle under two different conditions: (a) when it is affected by a gamma motoneuron and (b) when it is affected by an alpha motoneuron. It is shown that in the first case there is a delay of the output by about 12 ms whereas in the second case the system is delayed for about 37 ms.*

Keywords: *stationary point process, third-order spectral density function, phase estimation.*

1 INTRODUCTION

In this work, we present two algorithms for the estimation of the phase of a neuroelectric system of stationary point processes. The neuroelectric system is considered as a 'black box' where the incoming information (input) modifies its behaviour and produces a response (output). In this paper we shall consider that the output of the system is only known to us. The input and the output are stochastic signals called point processes, which are denoted by {M(t)} and {N(t)} respectively. In our example we shall assume that the two point processes are (i) stationary, (ii) orderly and (iii) strong mixing [1]. The point process ε(t) is assumed to be an additive Gaussian noise with stationary increments (see Figure 1) and the input is assumed to be a Poisson point process.

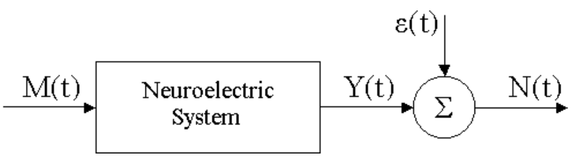

Figure 1. A graphical representation of the neuroelectric system.

The neuroelectric system that we examine is called muscle spindle and plays an important role in the initiation of the movement and the maintenance of the posture. The muscle spindle is considered as a stationary time-invariant and causal stochastic system. A stochastic model, which describes the linear relationship between the input and the output, is given by,

$$dN(t) = [a_0 + \int_{-\infty}^{t} a(t-u)dM(u)]dt + d\varepsilon(t)\,, \qquad (1)$$

where {dN(t)=N(t,t+dt]} and {dM(t)=M(t,t+dt]} are the increments of the input and output process respectively. By a(u) we denote the impulse response of a system. The constant a_0 represents the mean rate of the system when there is no input.

2 ESTIMATES OF THE FREQUENCY DOMAIN PARAMETERS

In this section we discuss the third-order spectral density function of a stationary point process and a method of estimating it. The estimate of the spectral density will be based on the modified third-order periodogram statistic, which is transformed properly in order to reduce its variance at constant value independent of the data stretch.

The third-order spectral density function of {N(t)} is given by

$$f_{NNN}(\lambda,\mu) = (2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{-i(\lambda u_1 + \mu u_2)\}$$
$$\times \mathrm{cum}\{dN(t+u_1), dN(t+u_2), dN(t)\}/dt$$

for $-\infty < \lambda, \mu < \infty$ (2)

where

$$\mathrm{cum}\{dN(t+u_1), dN(t+u_2), dN(t)\} = q_{NNN}(u_1,u_2)du_1du_2dt$$

is the third-order cumulant density of the output, for $u_1 \neq u_2, u_1 \neq 0, u_2 \neq 0$.

The finite mean corrected Fourier-Stieltjes transform of the point process is defined as

$$\hat{d}_N^{(T)}(\lambda) = \int_0^T h(t/T)\exp(-i\lambda t)[dN(t) - \hat{p}_N dt], \quad (3)$$

where $\hat{p}_N = N(T)/T$ is the estimate of the mean intensity of the point process. The function $h(u), -\infty < u < \infty$ is called a data window or a taper and is bounded, of bounded variation and vanishes for $|u| > 1$. More details about these functions and their definitions are given in [2],[3].

The transformed third-order periodogram is given by

$$\tilde{I}_{NNN}^{(T)}(\lambda,\mu) = T^{-1/2}(2\pi)^{-2} \times (H_3^{(T)}(0))^{-1}\hat{d}_N^{(T)}(\lambda)\hat{d}_N^{(T)}(\mu)\hat{d}_N^{(T)}(-\lambda-\mu) \quad (4)$$

where $H_3^{(T)}(\lambda)$ is the finite discrete Fourier transform of the data window.

In practice, in order to be able to use the FFT algorithm in the computation of the periodogram, we approximate (3) by the following expression:

$$\hat{d}_N^{(T)}(\lambda) \approx \sum_{t=0}^{T-1} h(t/T)\exp(-i\lambda t)[N(t+1) - N(t) - \hat{p}_N] \quad (5)$$

For more details on this approach refer to [4].

The estimate of the third-order spectral density function is now given by

$$f_{NNN}^{(T)}(\lambda,\mu) = (2m+1)^{-2} \times \sum_{j=-m}^{m}\sum_{k=-m}^{m} \tilde{I}_{NNN}^{(T)}(\lambda + 2\pi j/T, \mu + 2\pi k/T) \quad (6)$$

The properties of this estimate are discussed in [5]. Since $f_{NNN}(\lambda,\mu)$ is a complex function, its argument can also be estimated which is used for the recovery of the system's true phase. This can be done by using two different algorithms.

3 PHASE RECOVERY

In this section we present two algorithms for the recovery of the phase φ(λ) of the neuroelectric system. The phase here corresponds to the argument of the transfer function of the system. The transfer function, which is the one-sided Fourier transform of a(u), is given by

$$A(\lambda) = \int_0^\infty a(u)\exp(-i\lambda u)du, \quad -\infty < \lambda < \infty \quad (7)$$

Theorem: Suppose that the neuroelectric system can be described by the stochastic model given in (1), where the input, the output and the noise process satisfy the assumptions discussed above. Then the third-order cumulant density and the third-order spectral density are given by

$$q_{NNN}(u_1,u_2) = q_{YYY}(u_1,u_2) = p_M \int_0^\infty a(t)a(t+u_1)a(t+u_2)dt$$

for $u_1 \neq u_2, u_1 \neq 0, u_2 \neq 0$ (8)

and

$$f_{NNN}(\lambda,\mu) = \frac{p_M}{(2\pi)^2} A(\lambda)A(\mu)\overline{A(\lambda+\mu)} \quad (9)$$

where $p_M dt = E\{dM(t)\}$ is the mean intensity of the Poisson process.

If $A(\lambda) = |A(\lambda)|\exp\{i\phi(\lambda)\}$ and $f_{NNN}(\lambda,\mu) = |f_{NNN}(\lambda,\mu)|\exp\{i\theta(\lambda,\mu)\}$, then

$$|f_{NNN}(\lambda,\mu)| = \frac{p_M}{(2\pi)^2}|A(\lambda)||A(\mu)||A(\lambda+\mu)| \quad (10)$$

and

$$\theta(\lambda,\mu) = \phi(\lambda) + \phi(\mu) - \phi(\lambda+\mu) \quad (11)$$

Algorithm 1.

Let $\mu = \Delta\lambda$. Then

$$\lim_{\Delta\lambda\to 0}\frac{\theta(\lambda,\Delta\lambda)}{\Delta\lambda} = \lim_{\Delta\lambda\to 0}\frac{[\phi(\Delta\lambda)-\phi(0)]-[\phi(\lambda+\Delta\lambda)-\phi(\lambda)]}{\Delta\lambda} = -\phi'(\lambda)+\phi'(0)$$

Since $\int_0^\lambda(\phi'(\lambda)-\phi'(0))d\lambda = \phi(\lambda)-\lambda\phi'(0)$, we have

$$\phi(\lambda) = -\int_0^\lambda \lim_{\Delta\lambda\to 0}\{\theta(\lambda,\Delta\lambda)/\Delta\lambda\}d\lambda + \lambda c$$

where $c = \phi'(0)$ is an unknown constant defined as

$$c = \{\phi(\pi) + \int_0^\pi \lim_{\Delta\lambda\to 0}\{\theta(\lambda,\Delta\lambda)/\Delta\lambda\}d\lambda\}/\pi$$

and $\phi(\pi) = \kappa\pi$, $\kappa \in \mathbb{Z}$.

If we set λ=κΔλ and φ´(0)=φ(Δλ)/Δλ for $\Delta\lambda \to 0$, it can be shown that

$$\phi(\lambda) - \lambda\phi'(0) \cong -\sum_{j=1}^{\kappa-1}\theta(j\Delta\lambda,\Delta\lambda)$$

which leads us to use

$$G^{(T)}(\lambda) = -\sum_{j=1}^{\kappa-1}\arg(f_{NNN}^{(T)}(j\Delta\lambda,\Delta\lambda)) \quad (12)$$

as an estimate of $\phi(\lambda)-\lambda\phi'(0)$.

Algorithm 2.

We set $\omega = \lambda + \mu$ in (11). Then

$$\sum_\omega \theta(\lambda,\mu) = \sum_\omega[\phi(\lambda)+\phi(\mu)-\phi(\lambda+\mu)]$$

$$\Rightarrow \sum_{\lambda=0}^{\omega}\theta(\lambda,\omega-\lambda) = \sum_{\lambda=0}^{\omega}[\phi(\lambda)+\phi(\omega-\lambda)-\phi(\omega)]$$

We now assume that Δλ=1, λ= i and ω = n. Thus the above expression becomes

$$\sum_{i=0}^{n}\theta(i,n-i) = 2\sum_{i=0}^{n-1}\phi(i) - (n-1)\phi(n)$$

By letting $S(n) = \sum_{i=0}^{n}\theta(i,n-i)$ we obtain the phase φ(n) as follows

$$\phi(n) = \left[2\sum_{i=0}^{n-1}\phi(i) - S(n)\right]\Big/(n-1), \quad n = 2,3,\ldots,N \quad (13)$$

where the initial condition

$$\phi(1) = \sum_{n=2}^{N}[S(n)-S(n-1)]\Big/n(n-1) + \phi(N)/N$$

is satisfied with $\phi(N) = \kappa\pi$, $\kappa \in \mathbb{Z}$.

In both algorithms the integer κ is chosen to ensure the continuity of the estimates (neighbouring values are as close to each other as possible). Detailed discussions of the two algorithms can be found in [6],[7].

4 EXAMPLES

In this section we present the estimate of the phase of the neuroelectric system under two different conditions: (a) when it is affected by a gamma motoneuron and (b) when it is affected by an alpha motoneuron. In both cases the phase of the system is recovered using both algorithms presented above. In the first case the estimate of the phase starts from 0 and is a straight line with decreasing values. This means that there is a delay of the output by about 12 ms. In the second case the estimate starts from –3.14 and is a straight line with increasing values. It is now clear that the system is delayed for about 37 ms, since it is known from neurophysiology that the alpha motoneuron blocks the response of the muscle spindle, which implies that the impulse response function is negative (the behavior of the system is inhibitory). These results are in agreement with previous work (see [8], [9]), where both the input and the output point processes were used. As a conclusion we can say that the algorithm 2 is superior and gives more accurate results than the algorithm 1.

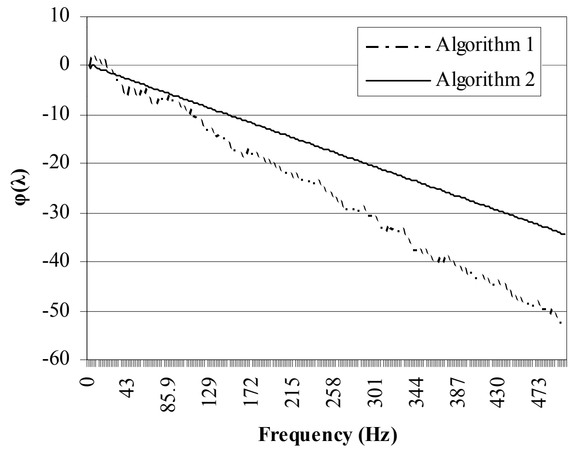

Figure 2. The estimate of the phase of the neuroelectric system when it is affected by a gamma motoneuron

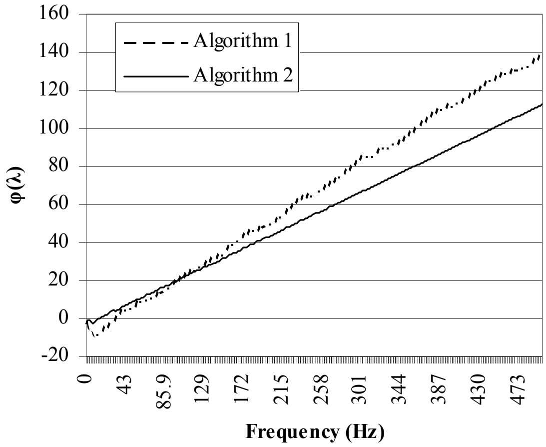

Figure 3. The estimate of the phase of the neuroelectric system when it is affected by an alpha motoneuron

5 CONCLUSIONS

In this paper we presented two algorithms for the estimation of the phase of a neuroelectric system involving stationary point processes. The estimate of the third-order spectral density function of the output was only used for the estimation of the phase. A quick way of estimating the spectral density is based on the third-order periodogram statistic. The results obtained by applying the algorithms to the two examples from the field of neurophysiology, showed that the algorithm 2 is superior and gives more accurate results than the algorithm1.

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