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Maximum Entropy Pattern Recognition Using Moment Descriptors

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Abstract

Geometric moments (GMs) have been successfully used to recognise image patterns in a number of applications. In this paper the information-theoretic method of Maximum Entropy (ME) is applied to the problem of two-dimensional pattern recognition from a finite set of geometric moments and compared to the Legendre Moments (LM) approach. Simulation results for image reconstruction (i) under noise-free conditions, (ii) of noisy images, and (iii) from noisy moment vectors demonstrate the superiority of the ME method, under which, even the undesirable 'blocking effect' produced by the block segmentation procedure is significantly reduced.

1. Introduction

The fundamental element used in many pattern recognition systems is the feature. A feature can be looked upon as a numerical value that quantifies some characteristic of an object that is unique to that particular type of object. For accurate and efficient recognition, the features used must be selected to allow distinction between object classes and be easily computed.

Moment descriptors of various forms have been extensively utilised as pattern features in many applications ranging from scene recognition to data compression and object matching [1-5]. Geometric, complex, Fourier-Mellin, radial and orthogonal moments [1,2,5,6,7,8,9] are some examples of moments.

Because of the wide applicability of image moments as features, many methods have been proposed for their fast computation [10,11,12] and for the VLSI implementation of moment-generating algorithms for real-time operation [13,14]

In 1980 Teague [6] suggested Legendre and Zernike moments (i.e. making use of Legendre and Zernike orthogonal polynomials) to recognise an image from a set of moments. However, despite its successful application to spectral estimation, image restoration, reconstruction and enhancement [15,16,17] there was no attempt for a ME solution to the above pattern recognition problem until very recently, when the first reported efforts [18,19] demonstrated the superiority of the ME method in producing results more accurate than the LM method while using fewer moments. The work presented here extends the comparative study to large images (using block segmentation) and to different types of noise on the images and on the moment vectors.

2 Background

The two-dimensional geometric moments (GMs) of order $(p+q)$ of the image intensity function $f(x,y)$ are defined as

$$M_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)\,dx\,dy \quad (p,q = 0,1,2,\ldots) \tag{1}$$

An important characteristic of moments is that they are global features. This means that all regions of the image affect the moment value to varying degrees.

When integrals are replaced by sums, Eq (1) gives the moments of order $(p+q)$ for a digitised image segment $f(x,y)$, i.e.

$$M_{pq} = \sum_x \sum_y x^p y^q f(x,y) \tag{2}$$

where $f(x,y)$ expresses the image gray-level.

A complete moment set (CMS) of order n consists of all the moments of order n and lower, i.e., $N_{total} = (n+1)(n+1)/2$ moment values.

The definition of the geometric moment, as given by Eq (1), has the form of the projection of the intensity function $f(x,y)$ onto the monomial $x^p y^q$. However, the basis set $\{x^p y^q\}$, while complete (Weierstrass approximation theorem [20]), is not orthogonal. Orthogonal moment forms [6] may be defined by using Legendre polynomial basis functions $P_n(x)$ [21] rather than conventional monomials. The Legendre moments can be easily related to other types of moments, see [9]. For example, an explicit relation between the LMs and the GMs is given by

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \sum_{j=0}^{p} \sum_{k=0}^{q} C_{pj} C_{qk} M_{jk} \quad (3)$$

A given Legendre moment depends only on geometric moments of the same order and lower, and conversely.

For the moments to be orthogonal, the image must be scaled to be within a 2 x 2 square centred at the origin, i.e. over the region $S = [-1,1] \times [-1,1]$.

By using Legendre rather than geometric moments, an approximate inverse transform - to obtain $f(x,y)$ from $\{L_{pq}\}$ - may be achieved by moment watching [6], i.e.

$$f_N(x,y) \simeq \sum_{j=0}^{N} \sum_{k=0}^{j} L_{j-k,k} P_{j-k}(x) P_k(y) \quad (4)$$

which is a truncated series, with N the maximum order of LMs available. It is clearly assumed, in this method, that all unknown moments ($n > N$) are zero. This not very legitimate assumption has actually instigated the use of the so successful maximum entropy formalism [22,23], which - with the irradiance distribution $f(x,y)$ considered as pdf - can be stated mathematically as:

Maximise

$$H = -\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \log f(x,y)\, dx dy \quad (5)$$

subject to

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x,y)\, dx dy = M_{pq} \quad (p+q = 1,2,\ldots,N) \quad (6)$$

and

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1 = M_{oo} \quad (7)$$

(scale normalisation). Solution of this standard variational problem yields

$$f_N(x,y) = \exp\left[-\lambda_o - \sum_{p,q} \lambda_{pq} x^p y^q\right] \quad (8)$$

where the λ's are Lagrange multipliers determined from the constraint conditions. The notation $f_N(x,y)$ refers to an estimate of the unknown intensity function $f(x,y)$ based on moments up to order N (i.e. on a CMS of order n).

The most important difference between the two methods is that, while the LM approach assumes all the unknown (or not given) moments to be zero, the ME method results in an estimate which is maximally non-committal with regard to missing information (i.e. unknown moments).

3. Performance Comparison

In this section, the two-dimensional, binary valued capitalised letter A shown in Fig 1, defined across a 21 x 21 pixel array, is used as the test case to simulate the performance of the two algorithms described in this paper.

3.1 Noise-Free Reconstruction

Fig 1 summarises the results for image reconstruction under noise-free conditions. Specifically, Figs 1(a) and 1(b) show reconstruction snapshots from different CMSs using both methods.

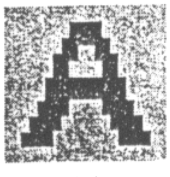
The normalised mean-square error (NMSE) between an image $f = f(x,y)$ and its reconstruction $\hat{f} = \hat{f}(x,y)$ from a finite set of its moments (up to order N), defined by

$$\bar{e}^{\,2}(N) = \| f - \hat{f} \|^2 / \| f \|^2 \quad (9)$$

has been adopted here, as a good measure of the image reconstruction ability of the moments, for comparing the performance of the two methods. Although the presented reconstructions are thresholded versions of the continuous ones, all mean square error calculations were developed by comparing the original and the actual reconstructed image (not the threshold image). Fig 1(c) shows this error as a function of N. This graph, together with the snapshots, demonstrates the superiority of the entropy method; the LM solution with moments up to even the 18th order (i.e. 190 moments) is not as good as the ME solution with moments up to only the 8th order (i.e. 45 moments).

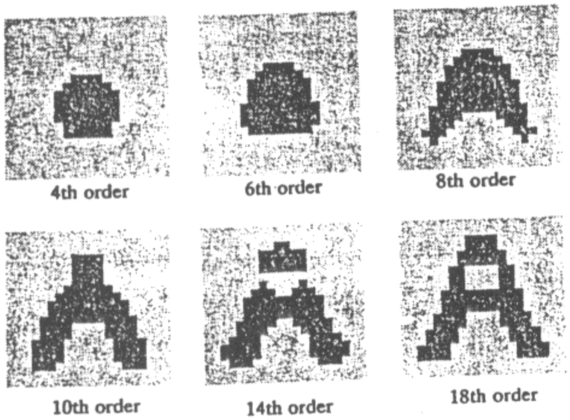
3.2 Reconstruction of Noisy Images

When the image $f(x,y)$ is corrupted by noise $n(x,y)$, the computed moment values, $\hat{M}_{pq}$, are expected to be erroneous, according to

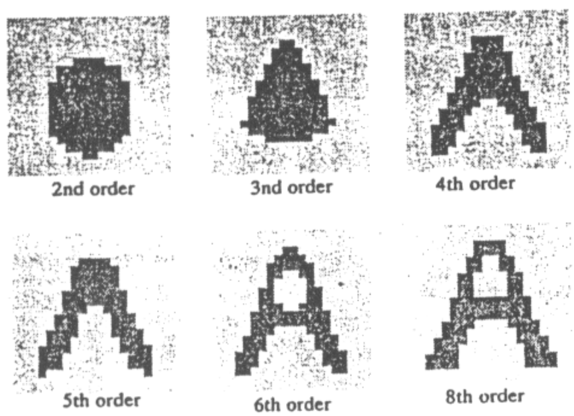


original letter

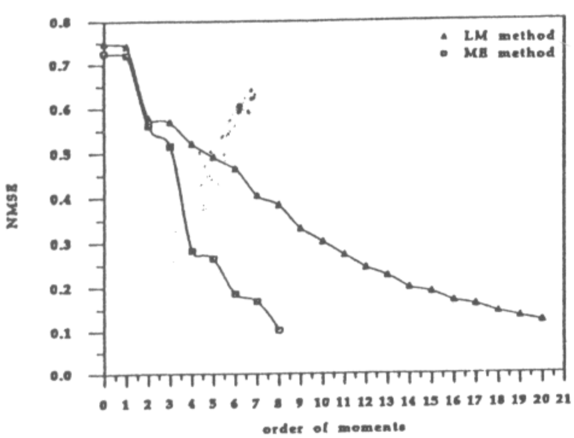
Fig.1 Noise-free reconstruction of the letter A using :
(a) the LM method
(b) the ME method



(a)



(b)



(c) Normalized reconstruction error

$$\hat{M}_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[f(x,y) + n(x,y)\right] x^p y^q \, dxdy \qquad (10)$$
$$= M_{pq} + {}^{n}M_{pq}$$

where ${}^{n}M_{pq}$ are the moments of the noise function. The observed moments are thus comprised of the 'deterministic' image moments and the 'random' noise moments. Since solution of this problem for a general type of noise is neither possible nor realistic, this study will be limited to simulations with two types of additive noise: normal and uniformly distributed. Fig 2 shows simulation results (no snapshots to save space) of the reconstruction of a noisy image (for the two types of noise and for different noise levels) by including increasingly higher-order moments.

As measure of the amount of noise present on the image, the signal-to-noise ratio (SNR) is used, which is defined here as the ratio of the image energy per unit area to the noise variance, i.e.

$$SNR \triangleq \left(\|f\|^2 / S_D\right) / \sigma_n^2 \qquad (11)$$

where S_D is the area of the region D, where f(x,y) is defined. The normalised reconstruction error (for LM and ME, respectively) as a function of N (the order of the CMS used) with the SNR as a parameter, was calculated by averaging over ten noisy image realisations generated for each SNR value. Comparing these plots, it turns out that, although the LM method appears to be affected by noise less than the ME method (particularly by normal noise), for reasonable noise levels the ME method is still superior to the LM method, i.e. the results from the two methods become very similar only for higher noise levels on the image, which is not of practical interest.

3.3 Reconstruction From Noisy Moment Vectors

The investigation of the effect of noise degradation in the image domain is followed by a study of the effect of noise degradation in the representation domain (i.e. noise on the moment vectors). Results for additive Gaussian and uniformly distributed noise on the moments are given in Fig 3. From the NMSE curves we come to a conclusion pertaining to both methods: For each SNR value, there is a certain optimal order of moments (different for the two methods and types of noise), which leads to the best image reconstruction. Using moments of order higher than the optimal order will result in larger reconstruction errors. This is clear at high noise levels in both methods, but certainly more striking (for any noise level and type) in the LM method.

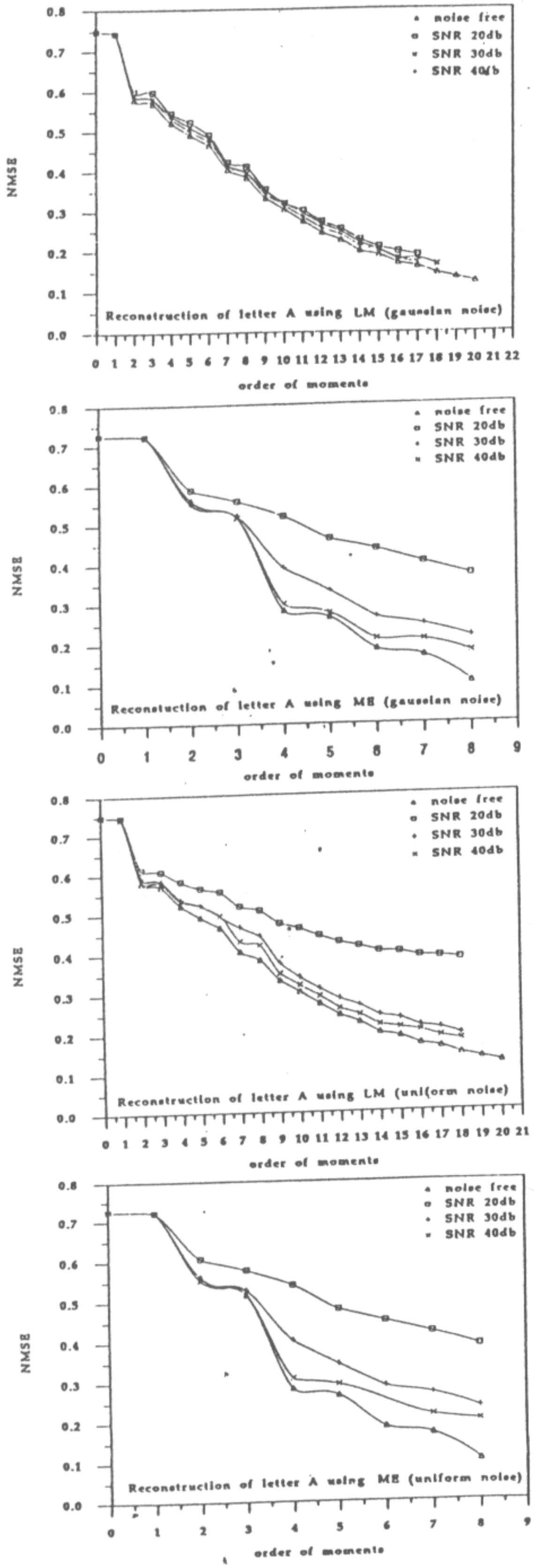


Fig.2 Reconstructing a noisy image

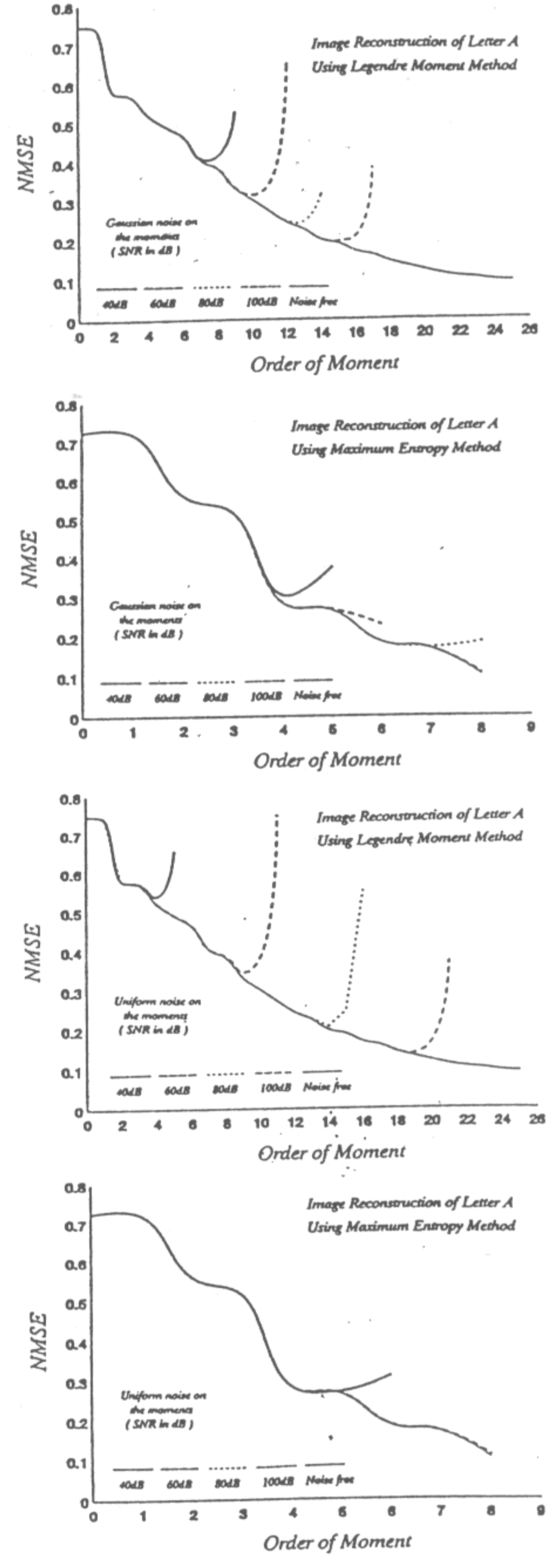


Fig.3 Reconstruction from noisy moment vectors

(a) (b) (c) (d) (e)

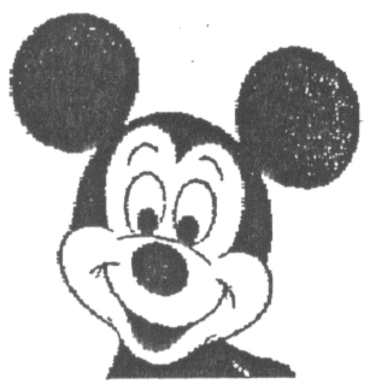
Fig.4 Noise-free reconstruction of a 60×60 pixels letter E (a) Original image

(b) Reconstruction with LM method on the whole image (moments up to 18_{th} order)

(c) LM reconstruction with block segmentation (moments up to 18_{th} order ; subimage size 20×20)

(d) Reconstruction with ME method on the whole image (moments up to 6_{th} order)

(e) ME reconstruction with block segmentation (moments up to 6_{th} order ; subimage size 20×20)



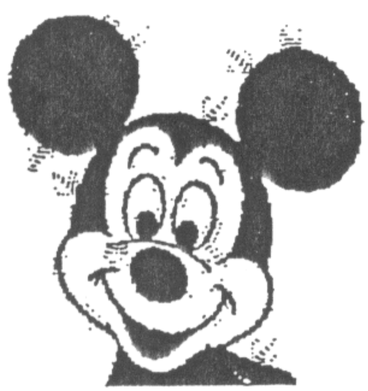
(a)

Fig.5 Reconstruction of a large (256×256 pixels) image with LM and ME method using block segmentation (16×16 pixels subimage)

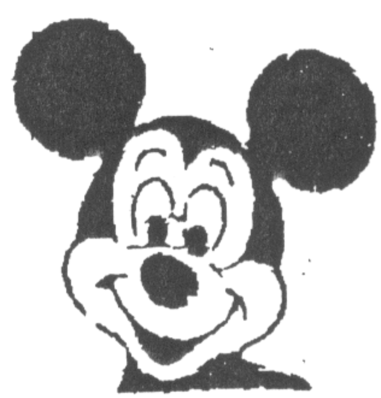
(a) Original image

(b) LM rec/tion (18_{th} order moments)

(c) ME rec/tion (4_{th} order moments)



(b)



(c)

4. Reconstruction of Large Images

In this section, the study is extended to large images and the two methods are applied on subpictures (using a block segmentation procedure), rather than on the whole picture. The improved result by operating on smaller blocks is initially shown in Fig 4, where we can also see the undesirable 'blocking effect' produced by the block segmentation procedure (due to discontinuities between subimages), which is present in the LM reconstruction (Fig 4(c)), but is almost non-existent in the ME reconstruction (Fig 4(c)). The second example pertains to a larger image (256 x 256 pixels Mickey), shown in Fig 5(a). The reconstructions under LM and ME are shown in Fig 5(b) and Fig 5(c), respectively, demonstrating once again the superiority of the entropic method, which manages to produce a good reconstruction with a minimal blocking effect using only moments up to the 4th order.

5. Concluding Remarks

From the simulation experiments presented here, it appears that the ME method is superior to the LM approach even under noisy conditions and when applied to large images. However, further study is necessary for gray-scale images.

The need for more computational time in the ME case becomes continuously less restrictive, because of the steadily increasing computing power available today and the development of fast algorithms [24,25].

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