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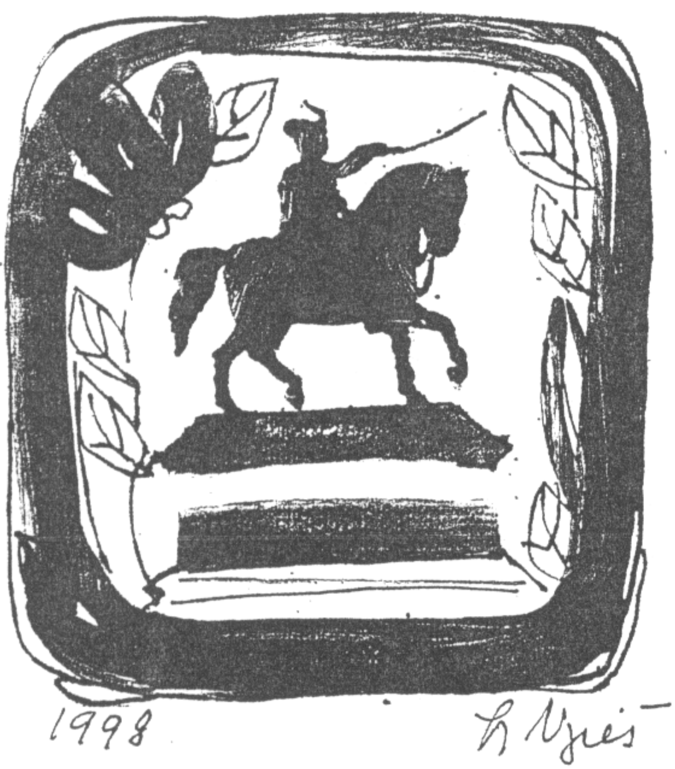

FAST COMPUTATION OF THE RADON TRANSFORM ON BLOCK REPRESENTED BINARY IMAGES

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Abstract: *This paper describes a binary image representation scheme, called Image Block Representation and presents an algorithm for the fast implementation of the Radon transform on block represented binary images. The main purpose of the Image Block Representation is to provide an efficient binary image representation that permits the execution of operations on image areas instead of image points.*

Key words: *Image Block Representation, Radon transform, fast algorithms*

1. INTRODUCTION

An advantageous representation for binary images, which is called *Image Block Representation* (IBR) and constitutes an efficient tool for image processing and analysis techniques, has been recently appeared [1]-[4]. The most important characteristic of the image block representation is that a perception of image parts greater than a pixel, is provided to the machine and therefore, all the operations on the pixels belonging to a block may be substituted by a simple operation on the block. Taking this feature into account, the implementation of new algorithms for binary image processing and analysis tasks, leads to substantial reduction of the required computational complexity. Using the block represented binary images, the fast computation of the Radon transform [5] is achieved.

2. IMAGE BLOCK REPRESENTATION

A bilevel digital image is represented by a binary 2-D array. Due to this kind of representation, there are rectangular areas of object value 1, in each image. These rectangulars, which are called *blocks*, have their edges parallel to the image axes and contain an integer number of image pixels. At the extreme case, the minimum rectangular area of the image is one pixel.

Consider a set that contains as members all the nonoverlapping blocks of a specific binary image, in such a way that no other block can be extracted from the image (or equivalently each pixel with object level belongs to only one block). This set represents the image without loss of information. It is always feasible to represent a binary image with a set of all the nonoverlapping blocks with object level. This representation is called *Image Block Representation* (IBR).

The IBR concept leads to a simple and fast algorithm, which requires just one pass of the image and simple ookkeeping process. In this pass all object level intervals are extracted and compared with the previous extracted blocks. A block represented image is denoted as:

$$f(x,y) = \{b_i : i = 0,1,...,n-1\} \quad (1)$$

where n is the number of the blocks. Each block is described by the coordinates of two corner points, i.e.:

$$b_i = (x_{1,b_i}, x_{2,b_i}, y_{1,b_i}, y_{2,b_i}) \quad (2)$$

where for simplicity it is assumed that: $x_{1,b_i} \leq x_{2,b_i}$ and $y_{1,b_i} \leq y_{2,b_i}$, as shown in Fig. 1.

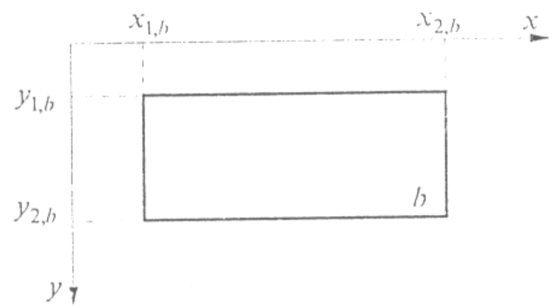

Figure 1. Each block b is described by the coordinates of its two corner points.

3. THE RADON TRANSFORM

The Radon transform, $g(s,\theta)$, of the image function $f(x,y)$ is defined as the projection (or the integral) of $f(x,y)$ along the line

$$s = x \cos\theta + y \sin\theta \quad (3)$$

oriented at angle θ counterclockwise from the positive y axis a distance s from the origin. The cost of computing the projections of an entire NxN image for a specific angle θ is $O(N^2)$ additions plus an overhead to decide which pixels hit the integration lines (3). In a lot of applications many such projections at different angles are required.

If $f(x,y)$ is unity inside a region occupied by a shape and zero elsewhere, then the image projections are the shape projections. Therefore in a block rerpesented image it is adequate to calculate the summations of the projections of the blocks along the integration lines (3) at a specific angle θ, as defined:

$$g(s,\theta) = \sum_{i=0}^{k-1} g_{b_i}(s,\theta) \quad (4)$$

3.1. Computation of the projections of one block

Consider the block b with coordinates (x_1, x_2, y_1, y_2). Then the projections of the block b along the lines (3) for a specific angle θ are shown in Fig. 2. For the computation of the projections the specific lines s where the four corners of the block belong are first calculated:

$$
\begin{aligned}
s_1 &= x_1 \cos\theta + y_2 \sin\theta \\
s_2 &= x_1 \cos\theta + y_1 \sin\theta \\
s_3 &= x_2 \cos\theta + y_1 \sin\theta \\
s_4 &= x_2 \cos\theta + y_2 \sin\theta
\end{aligned}
\qquad (5)
$$

For each s from $s_{min} = \min\{s_1, s_2, s_3, s_4\}$ to $s_{max} = \max\{s_1, s_2, s_3, s_4\}$ and from each and for each y from y_1 to y_2, the number of pixels that hit the line s is calculated and are added to the accumulator $g(s,\theta)$. For the calculation of the number of pixels n it is enough to determine the values x_{min}, x_{max} that correspond to the minimum and the maximum value of x that belongs to the specific line s for the specific y. These calculations are defined by:

$$
x_{min} = \begin{cases} (s - 0.5 - y\sin\theta)/\cos\theta, & x_1 \le (s - 0.5 - y\sin\theta)/\cos\theta \le x_2 \\ x_1, & x_1 > (s - 0.5 - y\sin\theta)/\cos\theta \\ x_2, & (s - 0.5 - y\sin\theta)/\cos\theta > x_2 \end{cases}
$$

$$
x_{max} = \begin{cases} (s + 0.49 - y\sin\theta)/\cos\theta, & x_1 \le (s + 0.49 - y\sin\theta)/\cos\theta \le x_2 \\ x_1, & x_1 > (s + 0.49 - y\sin\theta)/\cos\theta \\ x_2, & (s + 0.49 - y\sin\theta)/\cos\theta > x_2 \end{cases}
\qquad (6)
$$

$$
n = x_{max} - x_{min} + 1
$$

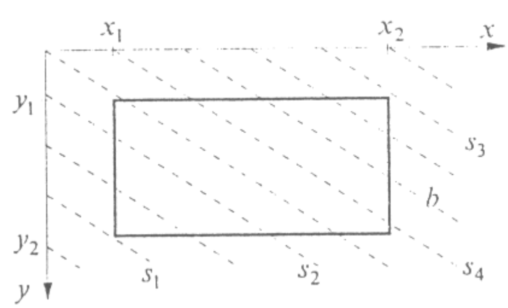

Figure 2. The projection lines that hit the block b for a specific angle θ.

3.2. Computation of the projections of the whole image

The computation of the projections of the whole image for a specific angle θ or for a set of angles $\{\theta_j\}$ takes place in terms of the computation of the projections of each block b_i, $i = 0,1,\ldots,k-1$. The computational time of the proposed algorithm is much less in comparison with the computational time of the algorithm that determines for each xel the corresponding line. Specifically the required time of the proposed algorithm is 3 times less for small images up to 100 times less for large 512x512 images in comparison with the algorithm that determines the corresponding line for each pixel of the image.

4. CONCLUSION

The image block representation may be seen as a physical model for the representation of binary images. Each block is represented by four integers, the coordinates of the upper left and lower right corner in vertical and horizontal axes. The image block representation is an information lossless process and therefore, from an information theory perspective, it is equivalent to the 2-D array image representation. Moreover, the image block representation is advantageous for image modelling. Usually, a block represented binary image requires considerably less storage space and therefore it is characterized by less entropy. The main advantage of the algorithm for the fast implementation of the Radon transform is that the complexity is independent of the image size, in contrast to implementations that are based on the 2-D array image representation, in the sense that the number of blocks is unchanged under image magnification.

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