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ORIGINAL RESEARCH

Real-time Computation of Krawtchouk Moments on Gray Images Using Block Representation

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Abstract
In the field of image analysis, the discrete orthogonal moments have better image representation capability than the continuous orthogonal moments and geometric moments. Krawtchouk moments are discrete orthogonal moments able to capture the local features of an image. The disadvantage of the Krawtchouk moments is the high computational cost which is increased as higher-order moments are involved in the computations. In this paper, we propose an effective approach for the computation of Krawtchouk moments. The gray image is decomposed in a set of binary images. The most significant binary images are represented using Image Block Representation and their moments are computed fast using block techniques. The least significant binary images are substituted by a constant ideal image called "half-intensity" image, which has known Krawtchouk moment values. The proposed method has low computational error, low computational complexity and under certain conditions, it is able to achieve real-time processing rates.

Keywords Krawtchouk moments · Image Block Representation · Real-time Computation · Image Analysis · Gray images

Introduction

Moments and moments functions have been widely used as features in image analysis and scene analysis [1–4], in stereo image matching [5], in image retrieval [6], image and object recognition [7–10] and image watermarking [11–13] applications. Other research fields where the moments can be used are classification [14], pattern recognition and 3D image recognition and understanding [15–17].

The geometric moment of order *(p,q)* of a 2-D function *g(u,v)* is defined as:

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(u,v)\, u^p v^q \, dudv$$

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The initially used image moments were based on geometric moments and their variations which are the central, normalized central and moment invariants sets [18]. The problem with the geometric moments is the large variations on the dynamic range of values and their numerical errors due to the approximation of the integrals in (1) with summations for digital images [18].

The problem above is solved by the use of continuous orthogonal functions as a basis set of moments. In this way, an image can be represented with no redundancy or information overlap between the moment values. Well-known continuous orthogonal moment sets are Zernike [19], Legendre [19], Fourier-Mellin [20]. However, some polynomials, such as the Zernike and Legandre polynomials, have some drawbacks. In particular, the Zernike polynomials are defined over the unit circle, while the Legendre polynomials are valid in the range [− 1, 1], thus the continuous orthogonal moments suffer from geometrical errors due to the required domain transformation and also from numerical errors due to the quantization processes [21].

The discrete orthogonal moments, which are based on discrete orthogonal polynomials, do not have digitization and coordinate space transformation errors, thus are capable of superior image representation. Well-known discrete orthogonal moment sets are the Tchebichef [22], the

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Krawtchouk [23], and the Hahn moments [24]. The Hahn moments are considered as a generalization of Tchebichef and Krawtchouk moments. TheTchebichef moments are able to capture the global features of an image, while the Krawtchouk moments capture local features [25]. The discriminative power of the moments has as a result the reconstruction of the image from a finite number of moment values [22, 23]. For this reason, moments have been used as features for image description and analysis. In fact, the Krawtchouk moments have a smaller reconstruction error than Zernike, Legendre and Tchebichef moments [26].

The order of the moments determines the type of image information that is described. The low-order moments represent the coarse features of the image; the high-order moments represent more detailed features, with increasing moment order, the representation detail of the image increases too.

The computation of moment requires vast computational effort, especially as the moment order increases. Various approaches have been proposed for the reduction of the computational effort. Some are based on the decomposition into rows and row segments on binary images [27–29]. Spiliotis and Mertzios [30] proposed a real-time method of geometric moment computation on binary images, which is based in an innovative image representation called Image Block Representation (IBR). Papakostas et al. [26], proposed the Image Slice Representation (ISR) for gray images which is used for the computation of discrete orthogonal moments. Spiliotis and Boutalis [30] proposed an extension of the IBR for gray images, which permits the real-time computation of geometric moments in gray images.

In this paper, we exploit the rationale introduced in [30] and propose an approach for the fast computation of Krawtchouk moments. These moments are discrete orthogonal moments capable of capturing the local features of an image, but they require a high computational cost, especially when higher-order moments are involved in the computations. The gray image is decomposed in a set of binary images; the most significant binary images are represented using Image Block Representation (IBR) and their moments are computed fast using block techniques. The moments of the remaining least significant binary images are replaced by surrogates that have been computed on a constant ideal image called "half-intensity" image, which has known Krawtchouk moment values. The proposed method has low computational error, low computational complexity and under certain conditions, it is able to achieve real-time processing rates.

The rest of the paper is organized as follows. In the next section, the image block representation for binary and gray images is reviewed. Next, the direct computation of Krawtchouk moments on binary images is reviewed, followed by their fast computation using block representation. After that, the computation of Krawtchouk moments on grayscale images is introduced using the Image Slice representation (ISR) method and the proposed method, which is based on the decomposition of the gray image in bitplanes. Experimental results follow that show off the superiority of the proposed method in respect to computation time compared to both of the direct method and the state-of-the-art ISR. The last section is devoted to the conclusions.

Block Representation of Binary and Gray Images

Block Representation of Binary Images

In a binary image, the pixels with object level are represented by a set of non-overlapping rectangles. Their edges are parallel to the axes, in such a way that every object pixel belongs to only one rectangle. These formed rectangles are called blocks and this representation is called Image Block Representation (IBR). The following definitions clarify the IBR.

Definition 1 A block is called a rectangular area of the image with edges parallel to the axes of the image, containing pixels with value 1, i.e. object level luminance.

Definition 2 A binary image is represented by blocks if it is represented by a set of non-overlapping blocks, and each image pixel with value 1 belongs to one and only one block.

The IBR process as described in Algorithm 1, is a fast process without numerical computations and requires one image scan and simple pixel checking operations.

Algorithm 1 Image Block Representation [30, 31].

Step 1: Consider each line *y* of the image f and find the object level intervals in line *y*.

Step 2: Compare intervals of line *y* with blocks of line *y*-1.

Step 3: If an interval does not match with any block, this is the beginning of a new block.

Step 4: If a block matches with an interval, the end of the block is in the line *y*.

A binary image represented by blocks described as $f(x, y) = \{b_i : i = 0, 1, ..., k-1\}$, where k is the number of the blocks and b_i is the *i-th* block that is described by the coordinates of two opposite diagonal angular points as:

$$b_i = \left(x_{1,b_i}, x_{2,b_i}, y_{1,b_i}, y_{2,b_i}\right)$$

Image Block Representation of Gray Images

Consider a gray image with intensity function $g(x,y)$, dimensions NxM and 2^n gray levels. The gray image can be decomposed into n binary images; each binary image is a bitplane of the original gray image. In other words, the pixel values of each binary image are derived from the bits of the same significance of the values of the corresponding pixel of the gray image. The first binary image is composed of the most significant bits (MSB) of the pixel values of the gray image g and is defined as $p_{n\text{-}1}$, the second most important is defined as $p_{n\text{-}2}$, while the bitplane with the least significant bits (LSB) is defined as p_0. The relation between the gray image $g(x,y)$ and the n binary images is:

$$g(x, y) = 2^{n-1}p_{n-1}(x, y) + \ldots + 2^1p_1(x, y) + 2^0p_0(x, y).$$

In Fig. 1, a sample of test gray images is demonstrated. Figure 2 demonstrates the decomposition of the test gray image of Fig. 1e according to the above rationale [30]. The test image of Fig. 2a which has 256 Gy levels, is decomposed to the 8 corresponding binary images. It can be observed that lower-order binary images look quite noisy. This feature is exploited in the sequel for the reduction of the computational cost of moment calculations. The n binary images resulting from the decomposition of the gray image can be represented by blocks.

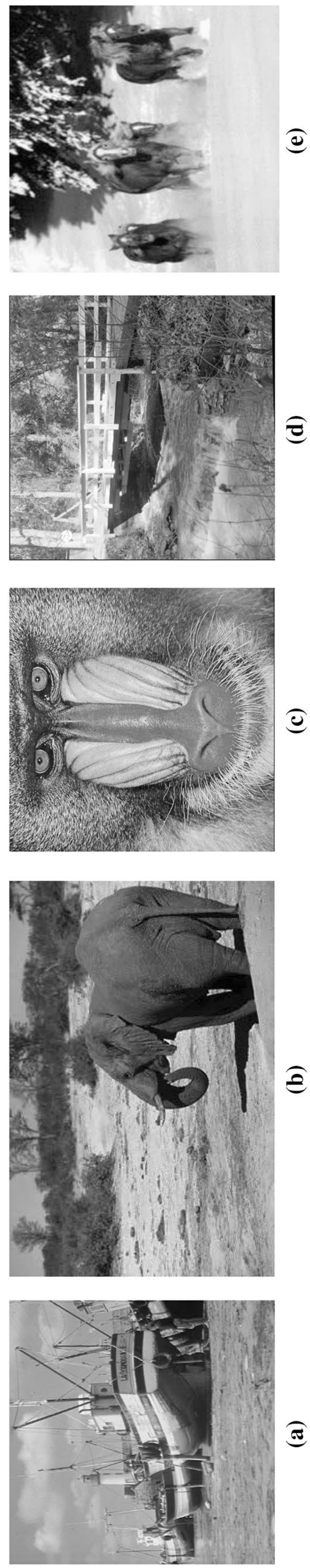


Fig. 1 A sample of test gray images used in experiments **a** Boat with size 256×256, **b** Elephant with size 384×256, **c** Baboon with size 512×512, **d** Bridge with size 512×512 and **e** Horses with size 1024×1024

Fast Computation of Krawtchouk Moments on Binary Images

Direct 2-D Computation of Krawtchouk Moments on Binary Images

The 2-D Krawtchouk moment of order pq of an image intensity function $f(x,y)$ with size NxM is defined as:

$$Q_{pq} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} K_p(x;s_1, N)K_q(y;s_2, M)f(x, y) \tag{4}$$

where $K_p(x;\, s_1, N)$ is the p-*th* order orthogonal Krawtchouk polynomial with respect to x axis, which is defined by the following recursive relation:

$$s_1(N - x + 1)K_p(x;s_1, N) = A(Ns_1 - 2xs_1 + 2s_1 + x - 1 - p)$$
$$K_p(x - 1;s_1, N) - B(x - 1)(1 - s_1)K_p(x - 2;s_1, N) \tag{5}$$

where

$$K_p(0;s_1, N) = \sqrt{\frac{s_1^p(1 - s_1)^{N-p}N!}{p!(N - p)!}}$$
$$K_p(1;s_1, N) = (Ns_1 - p)\sqrt{\frac{1}{Ns_1(1 - s_1)}}K_p(0;s_1, N) \tag{6}$$

Also the parameters A and B are defined as:

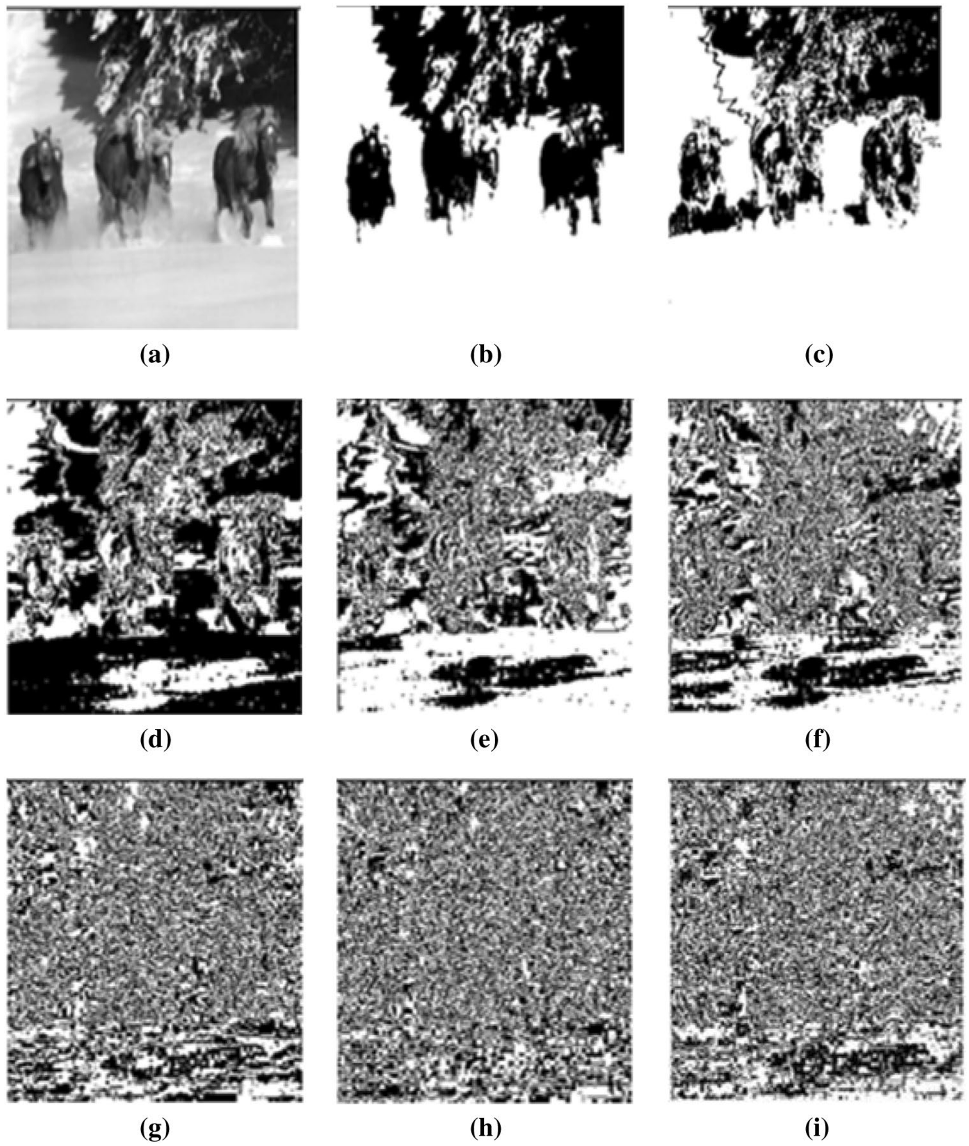


Fig. 2 Decomposition of a gray image Horses with size 1024 × 1024 and 256 Gy levels into 8 binary images. **a** The original gray image. **b**–**i** The binary images p_7 at **b** derived from the most significant bits, and p_0 at **i** derived from the least significant bits

$$A = \sqrt{\frac{(N-x+1)s_1}{x(1-s_1)}}$$
$$B = \frac{s_1}{1-s_1}\sqrt{\frac{(N-x+1)(N-x+2)}{x(x-1)}} \tag{7}$$

The polynomial $K_q(y; s_2, M)$ of q-th order with respect to y axis is calculated in the same way.

The computational complexity of Krawtchouk polynomials is reduced taking into account the symmetry property [32], which is demonstrated in Fig. 3 and defined as:

$$K_p(x;s_1,N) = (-1)^p K_p(N-x;s_1,N) \tag{8}$$

The calculation of Krawtchouk polynomials from (6) for large values of N leads to extreme values that produce numerical instabilities, as proved in image reconstruction from its computed moment values. Using input images with dimensions up to 2048 × 2048, the Krawtchouk polynomials and the moments are calculated correctly.

In addition, the parameters s_1 , $s_2 \in (0, 1)$ are decimals numbers and defined as two factors that determine the position and displacement of the Krawtchouk polynomials in the image. In this way, certain image features can be extracted. If $s_1 < 0.5$, then the polynomials are shifted to the left, if $s_1 > 0.5$, then the polynomials are shifted to the right of the image. If $s_2 > 0.5$, then the polynomials are shifted to the top of the image and if $s_2 < 0.5$, then the polynomials are

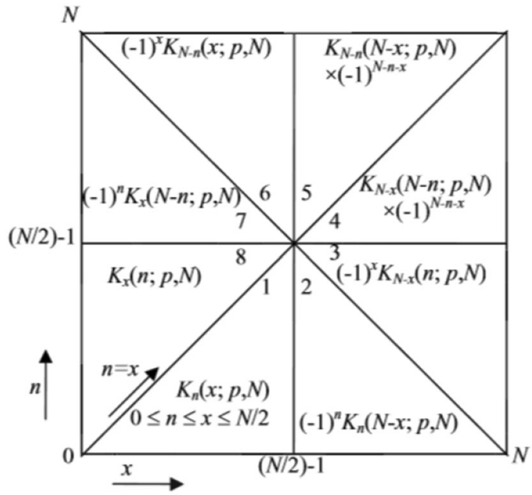


Fig. 3 Symmetric properties of Krawtchouk polynomials

shifted to the bottom of the image. If $s_1 = s_2 = 0.5$, then the polynomials are in the center of the image.

An image with size $N \times M$ can be reconstructed from a set of Krawtchouk moments up to the order $P \times Q$, using the following relation:

$$f(x,y) = \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} Q_{pq} K_p(x;s_1,N) K_q(y;s_2,M) \tag{9}$$

where $x = 0, 1, 2, ..., N-1$, $y = 0, 1, 2, ..., M-1$ and $P \le N, Q \le M$. If the number of moments used for reconstruction is equal to the number of image pixels, i.e., $P = N, Q = M$, then the reconstructed image is identical to the original image.

Fast Computation of Krawtchouk Moments on Block Represented Binary Images

In a binary image, assume that the brightness of the object is set to 1 and the brightness of the background is set to 0. As a result, only the pixels that describe the object will take part in the calculation of the moments. Thus, the Krawtchouk moments of a block represented binary image can be defined as:

$$Q_{pq} = \sum_{x} \sum_{y} K_p(x;s,N) K_q(y;s,M) \;\; \forall x,y : f(x,y) = 1 \tag{10}$$

Since all the pixels of image $f(x,y)$ with value 1 belong to the k blocks, the above Eq. (10) rewritten as:

$$Q_{pq} = \sum_{i=0}^{k-1} \sum_{x=x_{1,b_i}}^{x_{2,b_i}} \sum_{y=y_{1,b_i}}^{y_{2,b_i}} K_p(x_k;s,N) K_q(y_k;s,M) \tag{11}$$

Exploiting the rectangular form of the blocks with edges parallel to the image axes, the Krawtchouk moments of a block b are calculated as follows:

$$\begin{aligned} Q_{pq}^{b} &= \sum_{x=x_{1,b}}^{x_{2,b}} \sum_{y=y_{1,b}}^{y_{2,b}} K_p(x;s,N) K_q(y;s,M) \\ &= K_p(x_{1,b};s,N) \sum_{y=y_{1,b}}^{y_{2,b}} K_q(y;p,M) + \ldots + K_p(x_{2,b};s,N) \sum_{y=y_{1,b}}^{y_{2,b}} K_q(y;s,M) \\ &= \sum_{x=x_{1,b}}^{x_{2,b}} K_p(x;s,N) \sum_{y=y_{1,b}}^{y_{2,b}} K_q(y;s,M) \end{aligned} \tag{12}$$

The double sum of Krawtchouk polynomials is rewritten as the product of two separate sums, each sum containing the polynomial terms for the horizontal and vertical axis of the block, respectively. Using (12), Eq. (11) is rewritten as:

$$Q_{pq} = \sum_{i=0}^{k-1} \sum_{x_k=x_{1,b_i}}^{x_{2,b_i}} \overline{K}_p(x_k;p,N) \sum_{y_k=y_{1,b_i}}^{y_{2,b_i}} \overline{K}_q(y_k;q,M) \tag{13}$$

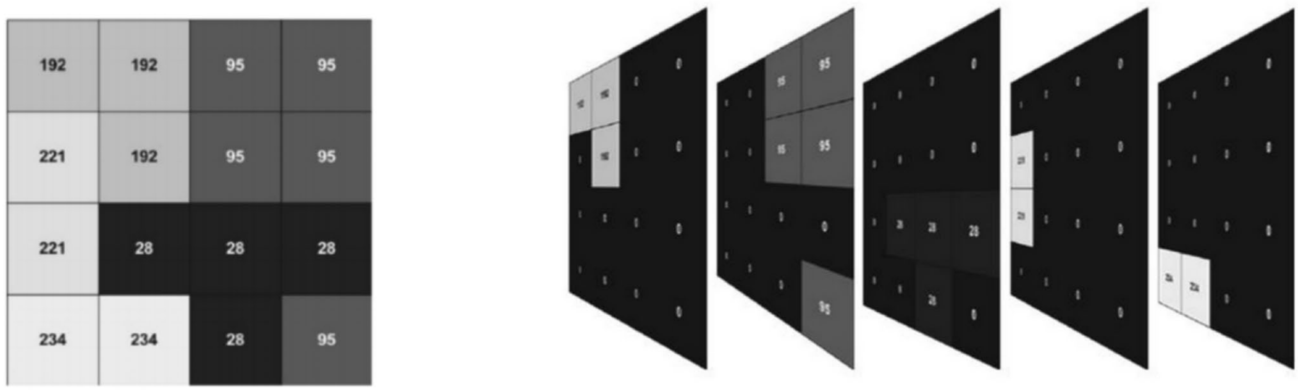
Fast Computation of Krawtchouk Moments on Gray Images

In this section, we present two alternative approaches, which use the idea of IBR for the computation of Krawtchouk moments on gray images. The first one is based on Image Slice Representation of gray images and was developed by the authors in [26]. We present this approach for comparison reasons. The second one is the proposed in this paper approach, which is based on the idea of decomposing the gray image in a set of binary images using the image bitplanes.

ISR Method for Computation of Krawtchouk Moments on Gray Images

For the fast computation of Krawtchouk moments on the grayscale images, Papakostas et al. proposed the Image Slice Representation (ISR) method [26]. The main idea behind the ISR is that a grayscale image consists of pixels with different intensities with values in the range of [0, 255], as it is shown in Fig. 4a.

Then, the gray image is decomposed into L binary image slices, where L is the maximum intensity value of



(a) (b)

Fig. 4 **a** Grayscale image with intensity [0,255] and **b** the ISR decomposition in 256 binary intensity slices

the image's pixels, as it is depicted in Fig. 4b. For example, if a grayscale image has 256 brightness levels, the number of slices that will be extracted is 256. Each image slice is unique and the pixels in it has only the values 0 or $p \in [1, 255]$.

Having decomposed the grayscale image into L binary slices, the IBR method can be used on each of them. The Krawtchouk moments of a grayscale image *f(x,y)* are computed by

$$\begin{aligned} T_{nm} &= \sum_{x-0}^{N-1}\sum_{y=0}^{N-1} t_n(x)t_m(y)\sum_{i=1}^{L} f_i(x,y) \\ &= \sum_{i=1}^{L}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t_n(x)t_m(y)f_i(x,y) \\ &= \sum_{i=1}^{L} f_i T_{nm}(i) \end{aligned} \tag{14}$$

where $T_{nm}(i)$ is the $(n+m)$-*th* order Krawtchouk moment of the *i-th* binary slice and can be computed with the IBR.

Where $f_i(x,y)$ is

$$\begin{aligned} &f(x,y) = \{f_i(x,y),\ i = 1,2,\ldots,L\} \\ &f_i(x,y) = \{b_{ij}, j = 0,1,\ldots,K_i - 1\} \end{aligned} \tag{15}$$

where b_{ij} is the *j-th* block of slice *i* and K_i is the number of image blocks having intensity f_i. Each block is described by the coordinates of the upper left and down right corner in vertical and horizontal axes.

The above procedure provides some speed up in the computation process, however, the acceleration in comparison with conventional 2D Krawtchouk moments calculation is not great.

Fast Computation of Krawtchouk Moments on Gray Images Using Bitplane Decomposition

The pixel values of a gray image are in range [0, 2^n-1] and the gray image consisted of n binary images. Substituting the Eq. (3) in Eq. (4), the following relation that connects the calculation of the Krawtchouk moments of the gray image and the n binary images is obtained by:

$$\begin{aligned} Q_{pq} &= \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} K_p(x;s,N)K_q(y;s,M)g(x,y) \\ &= \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} K_p(x;s,N)K_q(y;s,M) \\ &\quad \left[2^{n-1}p_{n-1}(x,y) + \ldots + 2^1 p_1(x,y) + 2^0 p_0(x,y)\right] \\ &= \left(2^{n-1}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} K_p(x;s,N)K_q(y;s,M)p_{n-1}(x,y) + \ldots \right. \\ &\quad \left. +2^0 \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} K_p(x;s,N)K_q(y;s,M)p_0(x,y) \right) \\ &= \left(2^{n-1}Qp_{(n-1)pq} + \ldots + 2^1 Qp_{1pq} + 2^0 Qp_{0pq}\right) \\ &= \sum_{i=0}^{n-1} 2^i Qp_{ipq} \end{aligned} \tag{16}$$

where $p_{n-1}(x,y), \ldots, p_1(x,y), p_0(x,y)$ are the binary images that compose the gray image $g(x,y)$ and $Qp_{(n-1)pq}, \ldots, Qp_{1pq}, Qp_{0pq}$ the Krawtchouk moments of these binary images which are calculated using IBR and Eq. (13).

The moments of the bitplanes p_i do not contribute equally to the gray image moments due to the weight factors 2^i, as it is observed from (16). Therefore, the less important binary images contribute less to the moments of the gray image. Moreover, the lower-order bitplanes look noisy with continuous black-and-white transitions, and are similar to a chessboard image or simply with an image with intensity 1/2.

Lemma 1 *The moment values of an image with intensity ½ are the half of the moment values of an image with intensity 1.*

Proof Exploiting the rectangular form of the images and according to (10), the Krawtchouk moment values of the "full intensity" image $f = 1, \forall x, y$ are $Qf_{pq} = \sum_{\forall x} K_p(x;s,N) \cdot \sum_{\forall y} K_q(y;s,M)$ and the moment values of the "half-intensity" image $h = 1/2, \forall x, y$ are

$$Qh_{pq} = \frac{1}{2} \sum_{\forall x} K_p(x;s,N) \cdot \sum_{\forall y} K_q(y;s,M) = \frac{Qf_{pq}}{2}$$

The approximated Krawtchouk moments $Q_{m,pq}$, by replacing the m least significant biplanes with the half-intensity image $h(x,y)$ is:

$$Q_{m,pq} = \sum_{i=m}^{n-1} 2^i Qp_{ipq} + \sum_{j=0}^{m-1} 2^j Qh_{pq} = \sum_{i=m}^{n-1} 2^i Qp_{ipq} + Qh_{pq} \sum_{j=0}^{m-1} 2^j \tag{17}$$

where Qh_{pq} are the Krawtchouk moments of the half-intensity image, which can be precalculated and used when required.

Representation Performance Evaluation on Gray Images

Replacing some lower-order bitplanes with the half-intensity image, then Eq. (17) leads to a reconstructed image $\hat{g}$ which is similar to the input image g, with a small approximation error that we can measure with the MSE metric:

$$\text{MSE} = \left(\frac{\sum_x \sum_y [g(x,y) - \hat{g}(x,y)]^2}{\sum_x \sum_y g^2(x,y)} \right)^{1/2}.$$

Experimental Results

To evaluate the performance of the proposed method, some experiments are carried out. The experiment evaluates the speedup of the Krawtchouk moments computation for gray images with the proposed method, in comparison to direct method (4) and to ISR method [26].

For the experimental evaluation, a computer with a total of 8 AMD Opteron cores at 2.2 GHz and 16 GB of memory was used. The operating system was Scientific Linux, all the programs implemented in C programming language, compiled with gcc for serial execution using one CPU core.

Experimental Results for Binary Images

In this section, we present the experimental results made by real measurements for the calculation of the Krawtchouk moments on binary images. It is noted that the Krawtchouk moment values computed using the proposed method in binary images are identical with the moment of the direct method without error.

(a) (b) (c)

Fig. 5 Set of binary images with size 1024×1024 pixels and the number of blocks k **a** Shapes with k = 1672 **b** Text Page with k = 18,753 **c** Chessboard with 10×10 pixel squares and k = 5304

Table 1 Time complexities (sec) and speedup values for the computation of the Krawtchouk moments from order 0×0 up to order $P\times Q$ for the test binary images using the direct and the proposed method with IBR and the achieved speedup

Image Size	1K×1K					
Order $P\times Q$	64×64			512×512		
Time (sec)	Direct	IBR	Speedup	Direct	IBR	Speedup
Shapes	114,12	0,47	242,81	7282,36	13,57	536,65
Text Page	114,24	2,39	47,80	7277,42	137,31	53,00
Chessboard	114,15	0,85	134,29	7273,89	39,55	183,92
Image Size	1920×1080					
Order $P\times Q$	64×64			512×512		
Time (sec)	Direct	IBR	Speedup	Direct	IBR	Speedup
Shapes	225,85	0,75	302,31	14,395,58	19,73	729,60
Text Page	225,41	2,97	75,84	14,406,84	163,72	88,00
Chessboard	225,78	2,68	84,26	14,414,38	137,88	104,54
Image Size	2K×2K					
Order $P\times Q$	64×64			512×512		
Time (sec)	Direct	IBR	Speedup	Direct	IBR	Speedup
Shapes	457,24	1,80	254,15	29,160,21	56,96	511,90
Text Page	455,90	2,49	183,45	29,186,29	127,43	229,04
Chessboard	456,93	3,30	138,29	29,168,23	161,87	180,19

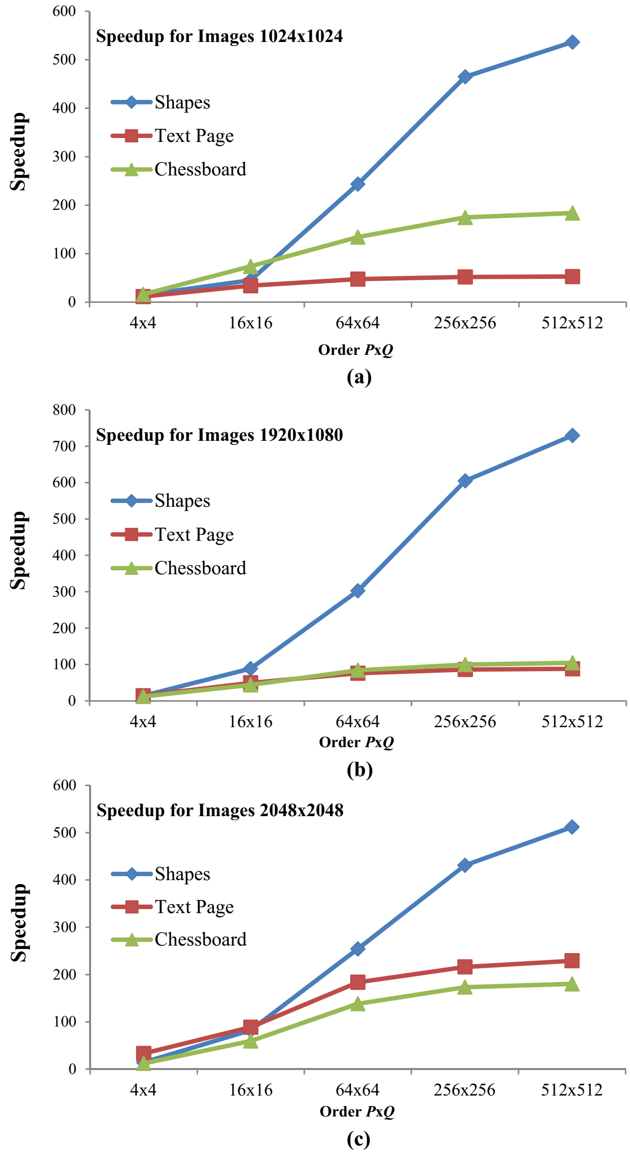


Fig. 6 Speedup values achieved using the proposed method for the Krawtchouk moment computation from order 0×0 up to order $P \times Q$, for the test binary images of Fig. 7 for sizes **a** $1K \times 1K$, **b** 1920×1080 and **c** $2K \times 2K$ pixels

The test binary images of Fig. 5 in different sizes have been used. For the size of 1024×1024, the number of blocks k is 1672 for the image “Shapes”, 18,753 for the image “Page” and 5304 for the image “chessboard” with a black-and-white transition every 10 pixels.

The execution times and the speedup values using the direct method of Eq. (4) and the proposed method of Eq. (13) for the Krawtchouk moment computation from order 0×0 up to the different maximum order are presented in Table 1. The execution time of the proposed method includes the time for the IBR and the time for moments computation. As it has already mentioned, test images’ dimensions are up to 2048×2048.

In Fig. 6, the speedup values achieved using the proposed are demonstrated. The execution time of the proposed method depends on the number k of the blocks of the image and the maximum moment order. It is observed that the achieved speedup values are very significant, ranging from 60 to 750.

Quality of Gray Image Reconstruction Using the Proposed IBR Moment Calculations

In this subsection, both subjective (optical) and objective evaluations of the reconstruction quality are provided, when the proposed IBR moment calculation is applied. In Figs. 7, 8 and 9, the reconstruction of images from the moments of (8-m) high-order bitplanes and the moments of m half-intensity planes are demonstrated. Figure 7a shows the input gray image g, while in Fig. 7b–h, the images $\hat{g}_m$ represented from the (8-m) most significant image bitplanes and m half-intensity planes of the input image. The image $\hat{g}_0$ is identical to g and is not demonstrated.

The replacement of the 4 or 5 least significant bitplanes with half-intensity images, results in the reconstructed images $\hat{g}_m$ the human vision system does not distinguish particular differences between $\hat{g}_m$ and the original input image g. Thus, it is expected that an identification system that uses the moments as features will classify patterns from the two images in the same class and this is a strong qualitative indication for the acceptance of the proposed method. So, for the computation of the moments, we can use only the first three or four bitplanes and replace the others with half-intensity images.

For the quantitative assessment of the quality loss, the MSE error values calculated using the metric of (18) between the image g and the images $\hat{g}_m$ are demonstrated in Table 3; it is observed that for $m = 4$ or 5, the error values are small and this substantiates the validity of the proposed

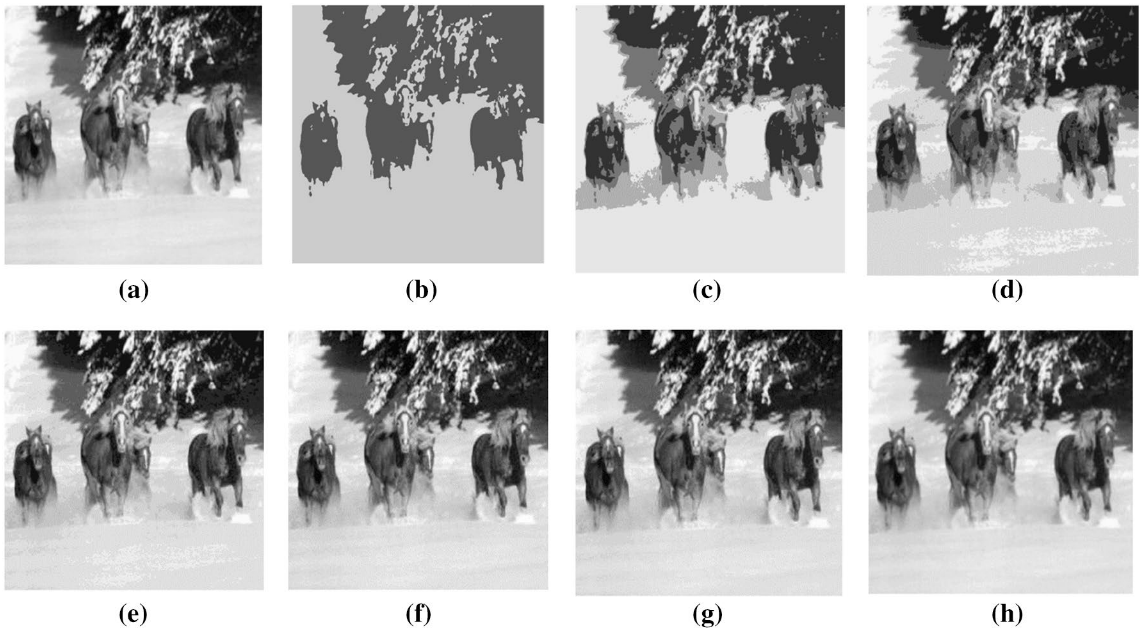


Fig. 7 **a** Original test image Horses with size 1024 × 1024 and **b–h** the reconstructed images from the (8-*m*) most significant image bitplanes and *m* half-intensity planes, where **b** *m* = 7, **c** *m* = 6, e.t.c. (h) *m* = 1. The case of *m* = 0 results in reconstructed image identical to (a)

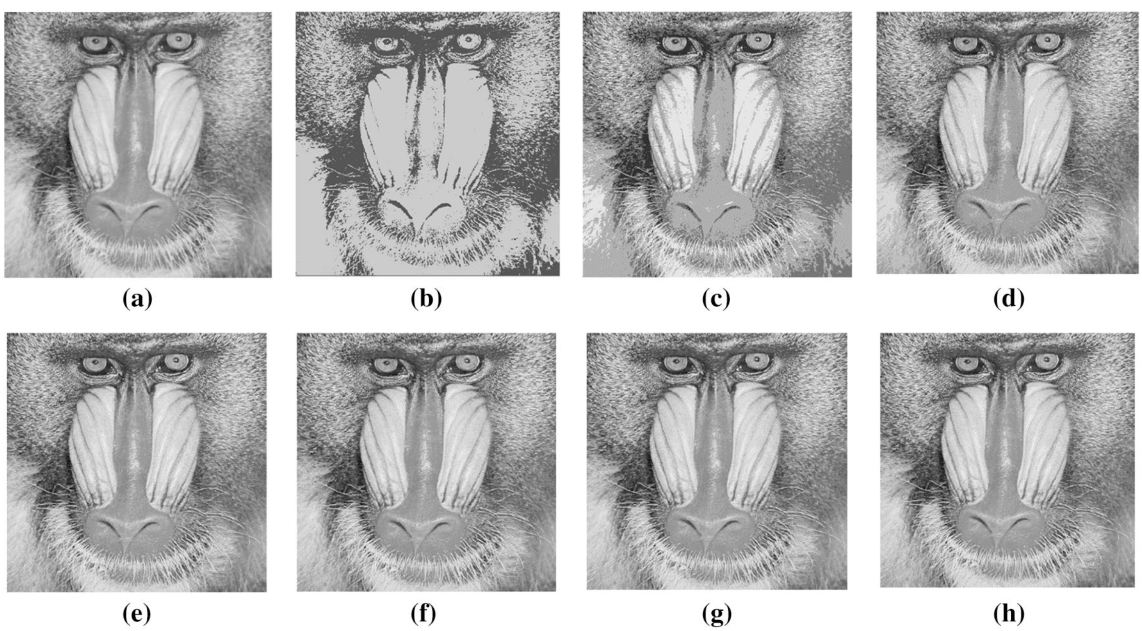


Fig. 8 **a** Original test image Baboon with size 512 × 512 and **b–h** the reconstructed images from the (8-*m*) most significant image bitplanes and *m* half-intensity planes, where **b** *m* = 7, **c** *m* = 6, e.t.c. **h** *m* = 1. The case of *m* = 0 results in reconstructed image identical to (**a**)

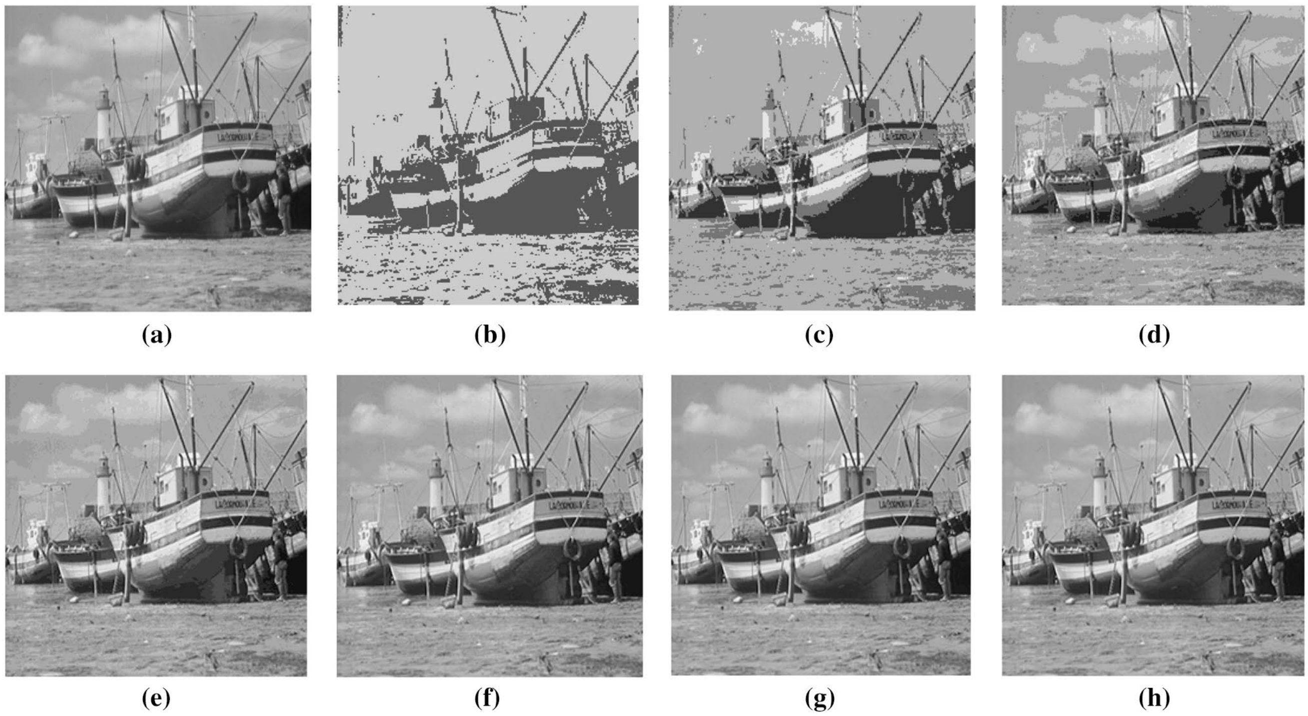


Fig. 9 **a** Original test image Boat with size 256×256 and **b**–**h** the reconstructed images from the (8-m) most significant image bitplanes and m half-intensity planes, where **b** $m=7$, **c** $m=6$, e.t.c. **h** $m=1$. The case of $m=0$ results in reconstructed image identical to (**a**)

Table 2 The number of blocks at each bitplane for some of the test images

Image	Size	p_7	p_6	IBR						ISR	
				p_5	p_4	p_3	p_2	p_1	p_0	Total number of blocks	Total number of pixels
Boat	256×256	2102	4152	6276	9800	12,917	14,754	14,968	14,931	57,291	65,536
Elephant	384×256	1922	5701	9997	16,264	20,123	21,762	22,203	22,443	87,667	98,304
Baboon	512×512	17,241	31,882	44,577	55,208	59,715	60,079	60,049	59,619	247,619	262,144
Horses	1024×1024	3896	11,814	27,430	52,806	86,740	122,731	165,026	184,534	570,666	1,048,576

As p_7 is defined the most important bitplane while p_0 the less important bitplane

Table 3 The MSE (%) between the original test image and the corresponding reconstructed images from (8-m) most significant image bitplanes and m half-intensity planes

Image	Size	$m=7$	$m=6$	$m=5$	$m=4$	$m=3$	$m=2$	$m=1$
Boat	256×256	27.57	13.75	6.88	3.53	1.76	0.92	0.53
Baboon	512×512	23.90	12.96	6.20	3.09	1.56	0.82	0.47
Horses	1024×1024	19.05	9.92	5.53	2.65	1.30	0.73	0.41

Table 4 Time complexities (sec) and speedup values for the computation of Krawtchouk moments for grayscale images from order (0,0) up to different orders, using the proposed IBR, the ISR and the direct method

Images with size 256×256							
Order of moments	Direct Method	ISR method		Proposed IBR method for $m=4$		Proposed IBR method for $m=4$	
	Time	Time	Speedup	Time	Speedup	Time	Speedup
4×4	0.0293	0.299	0.10	0.040	0.73	0.025	1.17
16×16	0.469	1.112	0.42	0.253	1.85	0.138	3.40
64×64	7.533	12.469	0.60	2.724	2.77	1.529	4.93
128×128	29.934	31.057	0.96	10.482	2.86	5.839	5.13
256×256	118.433	106.338	1.11	40.875	2.90	22.812	5.19
Images with size 512×512							
Order of moments	Direct method	ISR method		Proposed IBR method for $m=4$		Proposed IBR method for $m=4$	
	Time	Time	Speedup	Time	Speedup	Time	Speedup
4×4	0.116	1.344	0.09	0.205	0.57	0.135	0.86
16×16	1.877	5.021	0.37	1.524	1.23	0.977	1.92
64×64	29.909	39.847	0.75	18.495	1.62	11.523	2.60
256×256	477.344	447.297	1.07	281.759	1.69	174.825	2.73
512×512	1900.884	1727.689	1.10	1133.146	1.68	694.422	2.74
Images with size 384×256							
Order of moments	Direct Method	ISR method		Proposed IBR method for $m=4$		Proposed IBR method for $m=4$	
	Time	Time	Speedup	Time	Speedup	Time	Speedup
4×4	0.0654	0.4525	0.14	0.0759	0.86	0.043	1.52
16×16	0.703	1.2677	0.55	0.473	1.49	0.301	2.34
64×64	11.208	17.791	0.63	4.64	2.42	2.355	4.76
128×128	44.806	45.907	0.98	16.848	2.66	8.854	5.06
256×256	179.525	156.098	1.15	65.667	2.73	33.833	5.31
Images with size 1024×1024							
Order of moments	Direct method	ISR method		Proposed IBR method for $m=4$		Proposed IBR method for $m=4$	
	Time	Time	Speedup	Time	Speedup	Time	Speedup
4×4	0.469	4.731	0.10	0.277	1.69	0.167	2.81
16×16	7.556	13.359	0.57	1.409	5.36	0.731	10.34
64×64	119.498	76.531	1.56	13.26	9.01	6.234	19.17
256×256	1908.848	1050.847	1.82	187.03	10.21	85.032	22.45
512×512	7639.013	4070.947	1.88	707.243	10.80	323.254	23.63

The proposed method used for (8-m) most significant bitplanes and m half-intensity images, $m=4$ amd $m=5$. The time for IBR of the bitplanes included in the execution times of the proposed method

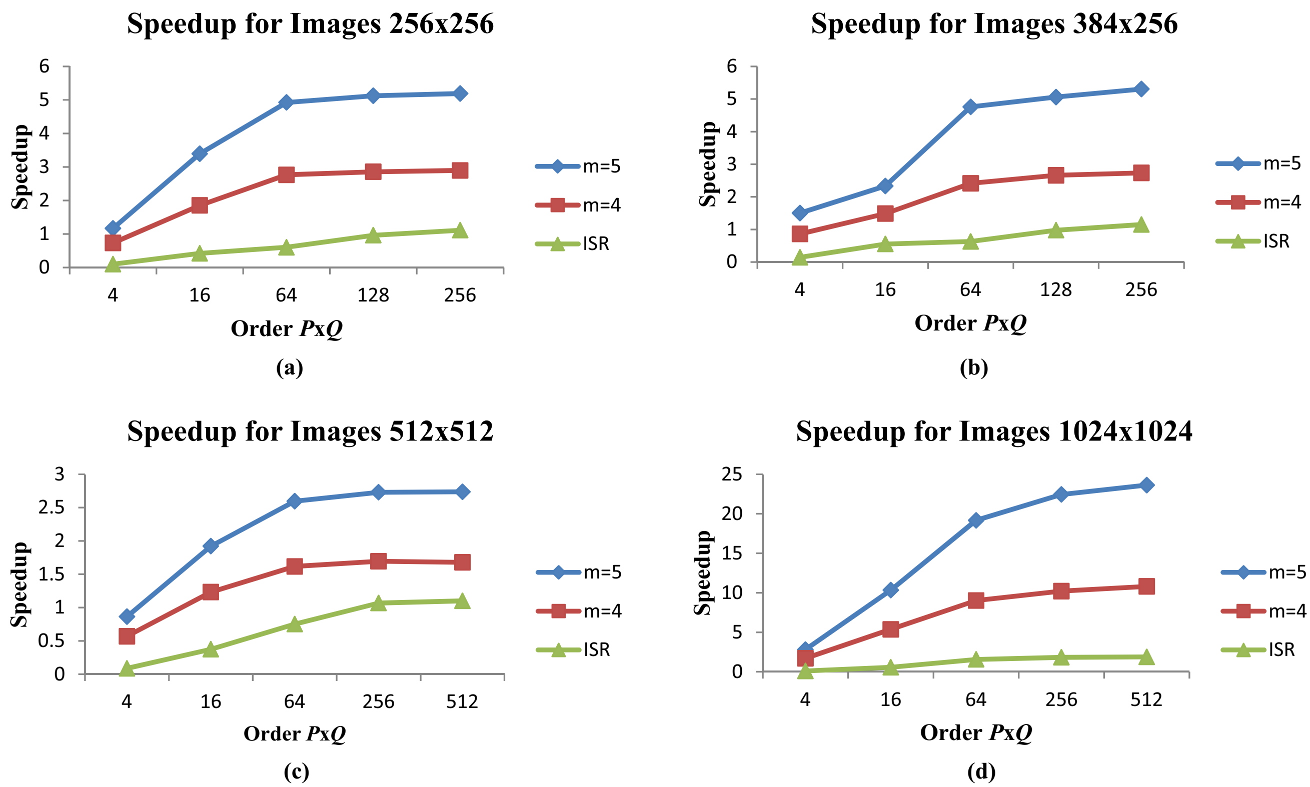


Fig. 10 The speedup values achieved for the computation of Krawtchouk moments from order 4×4 up to order $P \times Q$, using the proposed method for a number of gray test images of different sizes and for using (8-m) bitplanes and m half-intensity images, ($m=5$) and ($m=4$) and ISR method

method. Similar results have been obtained for all of the test gray images used in the experiments.

Speedup Gain Using the Proposed Method in Grayscale Images

In this subsection, we are testing the speedup gain of the proposed IBR method in comparison with the direct and the ISR based methods for Krawtchouk moments calculation. Table 2 demonstrates the total number of the pixels in the grayscale images of Fig. 1 and compares them with the number of blocks exported to each bitplane and the total number of blocks that extracted using the IBR and the ISR method, respectively.

It is observed that in IBR, the number of blocks increases as the significance of the bitplane decreases; since the 4 or 5 lower-order bitplanes are substituted by half-intensity bitplanes, the total number of blocks of 3 or 4 higher biplanes is significantly reduced. In ISR, the total number of blocks that are extracted is comparable with the number of pixels. Moreover, in ISR, the number L of binary images is 256, while in the proposed method, the number of bitplanes used n is 3 or 4.

Table 4 represents the computation time of Krawtchouk moments with the use of IBR and ISR methods for a number of test images with different sizes and the achieved speedup in relation with the 2D direct method. It is observed that the proposed method achieves better performance than the ISR method [26], due to the fact of substitution of lower-order bitplanes with half-intensity bitplanes.

In pattern recognition and image analysis applications, images with small size are usually used, where a small number of moments are utilized as features. From Table 4, it can be seen that under these conditions, the proposed method operates fast in rates that are real time or near to real time, where real time is defined by a video rate of 24 frames/sec.

Figure 10 demonstrates the computational time of Krawtchouk moments using these methods, for different gray images. The execution time of the proposed method and the other comparable ISR method include the time for the block representation of the binary images and the time for the computation of moments. In our method, the moments of the half-intensity image are calculated considering the whole image as one block as described in Lemma 1, with negligible computational load. According to the analysis of the error that was discussed in the previous subsection, it is qualitatively acceptable to use the images g_5, g_4 with 3 and 4 real bitplanes and 5 and 4 half-intensity images, respectively. For these images, it is observed that the proposed method is superior.

In general, the speedup value using the proposed method depends on a number of parameters, as: i) The image content and the number of blocks required for the representation of the higher-order bitplanes. The execution time of the proposed method increases with the number of blocks. ii) The number of the higher-order bitplanes that are used. iii) The image size, since the number of blocks does not vary significantly for similar images with different sizes. iv) The total number of moment values required, since the time for the execution of the block representation of the bitplanes is divided among a greater number of moments.

Conclusion

In this paper, a fast computation method of Krawtchouk moments in grayscale images and a brief description of fast calculation of Krawtchouk moments in binary images are presented. The method is based on the decomposition of the input gray image, with 8 bit/pixel, to the corresponding bitplanes which are represented using image blocks. The lower-order bitplanes look similar to an ideal image called half-intensity image with moment values equal to the half of full intensity image. Thus, it is adequate to compute only the moments of the higher-order bitplanes and replace the least significant bitplanes with half-intensity images. This results in significant acceleration while the error between the original image and the new reconstructed is acceptable.

Experimental results for gray images with 8 bit/pixel, show that the substitution of the lower 5 bitplanes with the half-intensity image, results in small error values in the reconstructed images and a significant acceleration of the computation of moments.

Moreover, under certain conditions usually met in pattern recognition applications requiring a moderate number of moment calculations in images of small size, the proposed method operates very fast, achieving real-time performance.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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