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SPH numerical simulation of surf zone characteristics

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ABSTRACT: Numerical simulation of wave breaking over relatively mild sloping beaches is pursued in this study with the implementation of a modern promissory method called Smoothed Particle Hydrodynamics. Validation of the method is attempted through the use and calibration of a recent free academic 'open source' code called SPHysics. Its various assumptions and parameters are adjusted to simulate experimental results. An effort is undertaken to confirm its ability to capture the near-shore wave breaking and consequent bore formation dynamics in the surf/swash zone. Inherent and calibration defects of the model are highlighted and particular upgrading measures are suggested for oncoming research.

1 INTRODUCTION

Coastal scientists and engineers are involved more and more nowadays in the environmental aspects of coastal hydraulics. In this framework, the description of the detailed near-shore wave pattern evolution plays one of the most significant roles in comprehending the extremely complex character of near-shore processes. Among those, wave propagation, shoaling and depth induced breaking are prominent. Especially the latter is of major importance in assessing the surf/swash zone characteristics, in quantifying the near-shore velocity and vorticity profiles, in deriving the undertow return-type flow as well as shoreward net drift-type motion and finally in defining the overall coherent and intermittent turbulent structures. All above primarily control coastal sediment movements, thus long- and cross-shore morphodynamic evolution. Moreover, they become important in describing the aeration and mixing processes that take place in the surf zone and, in combination with the sediment transport and the descending turbulent eddy formation, are responsible for the definition of quality and safety criteria for recreation and related activities. On top of that, the current climate change is often the major factor in provoking lastingly extreme weather and sea conditions and consequent risky inundation events, especially in coastal low-land beach formations. This relates immediately to the run-up on relatively mild-sloping coasts and e.g. respective sand dune delimitation and preservation or overtopping and breaching. Similar processes apply also to coastal protection works. Accordingly, near-shore wave breaking and associated turbulence in the surf, swash and run-up zones have been studied considerably both physically and numerically, throughout the last decades, yet the hydrodynamics involved in the respective description of the process are far from completely understood.

Detailed representation of more sophisticated features is needed nowadays, such as coherent turbulent structures, obliquely descending eddies (Nadaoka et al. 1989) under the breaker trough and horizontal vortices under the breaker crest, that primarily govern sediment suspension and cross-shore morphodynamic evolution. Characteristic measures of the above mentioned features have been extensively derived throughout the last decades by various physical modelling laboratory flume experiments of wave breaking, on relatively mild slopes. These efforts are amazingly large in number and mostly account for linear and non-linear regular waves.

On the other hand modern computational approaches comprise modelling the full Navier-Stokes (NS) equations in combination with averaging and surface tracking techniques like RANS-VOF or Sub-Grid Scale models for turbulence closure like LES-SGS. Another elaborate yet computationally very expensive method is DNS. Somewhere in the middle of those, the Smoothed Particle Hydrodynamics (SPH) method is discerned. It stands out as a promising modern technique in dealing with highly deformed free surface flows (e.g. plunging breakers), using pure Lagrangian formulation without the strenuous use of a toggling computational grid.

The primary scope of our ongoing investigation is the accurate simulation of the highly non-linear process of wave breaking on beaches with plane and relatively mild sloping shapes. Secondarily, we focus on quantifying the undertow and run-up on them, as well as the loading and overtopping of coastal structures with steeper slopes. Moreover, a detailed description of the turbulent features inside the surf/swash zone is to be pursued and all of the above, by means of a modern computational approach based on a meshless domain discretization. Application of the latter is based upon one of the most recent comprehensive laboratory experimental studies on near-shore breaking waves and consequent turbulence transport under them by Stansby & Feng (2005) [SF, hereafter]. Calibration of the various parameters of the method is being attempted and the key features are evinced through inter-comparisons of several implementations. Comparative analysis between the model and the experiment results are presented here, shedding some light to the limitations of the model application and indicating specific upgrades for prospective research.

2 METHODOLOGY

SPH is a mesh-free numerical technique for solving the NS equations, by discretizing the continuum domain in particle form. These particles represent, in practice, the nodal points, which constitute the computational frame and carry all scalar and vector field information needed. The method's Lagrangian approximation, combined with the use of integral interpolation smoothing functions for an arbitrary domain's variable $A(\mathbf{r})$ (Eq.1) and its derivative, enables one to treat complex phenomena in fluid dynamics, such as free surface flows with intense deformations, like wave breaking and wave-structure interaction. There is extensive literature on relevant simulations, worth noting among them those by Dalrymple & Rogers (2006) and Crespo et al. (2007). The basic relation of the SPH approximation technique reads:

$$A(\mathbf{r}) = \int A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}',h)d\mathbf{r}' \Rightarrow A(\mathbf{r}) = \sum_j A_j \left(m_j / \rho_j \right) W_{ij} \tag{1}$$

where m, ρ = particle mass and density, $\mathbf{r}$ = particle arbitrary location, $\mathbf{r}'$ = distance between particles i and j, $W(\mathbf{r},h)$ = a distance varied interpolation weighting function called 'kernel' and given analytically in various implemented versions (Gómez-Gesteira et al. 2007), such as Gaussian, quadratic, qubic B-spline, quintic etc. Probably the most important variable is the smoothing length h, which primarily controls the magnitude of the interpolation process throughout the domain and consequently the accuracy of computations. The choice of the aforementioned kernel is seemingly irrelevant, provided that certain essential attributes apply (Liu & Liu 2003; Makris et al. 2009), particularly for h and spacing Δx tending to zero.

Fundamental thorough references on the theory of SPH can be found in Monaghan (2005) and the textbook of Liu & Liu (2003). Additionally, essential concepts describing the grounds of our current research can be tracked in our previous work (Makris et al. 2009).

In this context an academic 'free', open-source code called SPHysics (Gómez-Gesteira et al. 2007) is implemented. Among its various assumptions, the code incorporates the classical, for the SPH literature, concept of an artificial empirical viscosity term Π_{ij} (Monaghan 2005) in the NS equations:

$$\Pi_{ij} = -a\mu_{ij}\bar{c}_{ij} / \bar{\rho}_{ij} \tag{2}$$

where $\alpha = 0.01 \sim 0.1$, $\mu_{ij} = (\mathbf{u}_i - \mathbf{u}_j)\, \mathbf{r}_{ij} / (\mathbf{r}_{ij}^2 + 0.01h^2)$, $\mathbf{u}$ = the velocity vector, c = the computational speed of sound and over-bared features denoting average property values between i and j particles. The empirical coefficient α is considered to be necessary for numerical stability, yet it may provoke excessive dissipative performance of the model. Furthermore the eddy viscosity assumption (Boussinesq hypothesis) is also employed in the framework of a standard Smagorinsky-type model for the derivation of turbulent eddy viscosity as $v_t = [\min (C_s\, \Delta l)]^2\, |\mathbf{S}_{ij}|$, where $\mathbf{S}_{ij}$ = the strain tensor and the Smagorinsky coefficient $C_s = 0.12$. This approach gives rise to the ultimate Sub-Particle Scale (SPS) stress tensor symmetric formulation, which can be traced through equations 10 ~ 12 in Makris et al. (2009). Moreover, the artificial compressibility approximation is also included in the model, with the appointment of an equation of state instead of an additional arduous Poisson-type equation for pressure (Monaghan 2005):

$$P = B\left[\left(\rho/\rho_o\right)^\gamma - 1\right] \tag{3}$$

where $\gamma = 7$, $B = c_o^2 \rho_o / \gamma$, reference density $\rho_o = 1000$ kgr/m^3, speed of sound $c_o = \partial P / \partial\rho|_{\rho o} = c_B \cdot V_{max}$, V_{max} = the maximum velocity in the computations and c_B = an artificial compressibility factor. By changing B, we can artificially modify the speed of sound to approach nearly incompressible conditions and speed up computations. Additionally, solid boundary conditions are dealt with as repulsive, based on the Lenard-Jones molecular potential (Monaghan & Kos 1999). Ultimately, the available numerical schemes comprise Predictor-Corrector (PC) type and Verlet (V) type algorithms (Gómez-Gesteira et al. 2007). A choice of constant and rather small time step Δt ensures fulfilment of the Courant-Friedrichs-Lewy criterion, yet increases the computational time especially for fine resolution simulations.

The applicability of the model and its potential calibration is being investigated by nesting to the present tests a numerical wave flume setup, simulating the SF experiment.

3 MODEL IMPLEMENTATION

3.1 *Numerical setup and experimental data*

The geometric and hydraulic features of the experiment (SF) used as input data for the numerical wave tank simulations undertaken for the validation of the SPHysics code are presented in Table 1. Further detailed description of the experimental setup can be found in SF.

3.2 *Calibrated numerical features*

The various test cases employed, according to the distinctive calibrated features are presented in Table 1. The basic, among them, appears to be the smoothing length $h = c_f\, (\Delta x^2 + \Delta z^2)^{1/2}$, where c_f = calibration coefficient, Δx, Δz = horizontal, vertical spacing respectively. Variation of its implementation and respective test numbers appear, as test series b in Table 1. Test a corresponds to the default calibration described in Makris et al. (2009). Types of kernel and numerical algorithm were also altered, in order to track their influence on the results and appear as test series c and d respectively. Further separate calibration through the viscosity treatment assumption is pursued, regarding SPS Smagorinsky-type model (a), laminar (i) and artificial viscosity, with the values of coefficient α shown by test series e in Table 1. Simulations were also conducted with finer Δx, Δz = 0.01 m, h = 0.92 ~ 1.52 m and $\Delta x/h$ = 0.7686 ~ 0.4652.

4 RESULTS COMPARISON AND DISCUSSION

Characteristic preliminary results provided by Makris et al. (2009) have shown either good qualitative or rather acceptable quantitative agreement between SPHysics simulations and SF experiment. Further calibration of the SPHysics model was expected to yield better results, some typical of which are given below. In a very recent study, De Padova et al. (2009) argue that decrease of h and simultaneous increase of the dimensionless spacing factor $\Delta x/h$, combined with the use of the cubic-spline kernel, give rise to drastic degradation of their results compared with experimental data. We presently affirm and corroborate their outcome, also for the use of quadratic kernel and proceed farther in our analysis of the spacing factor *Δx/h* influence providing an optimized value for our simulations, namely that of Test case b7 (Table 1).

Table 1. Test cases according to main calibrated features - Geomteric and hydraulic experimental features

Test	*h* (m)	*Δx* (m)	*Δx/h*	c_f	**Test**	Viscosity	**Test**	Kernel/Algorithm	**Test**	c_B
a	0.0260	0.02	0.7686	0.92	a	SPS	a	quadratic/PC	a	16
b	0.0305	0.02	0.6547	1.08	e	α=0.05	c	cubic-spline/PC	g	30
b2	0.0232	0.02	0.8623	0.82	e2	α=0.04	c2	Gaussian/PC	g2	40
b3	0.0288	0.02	0.6932	1.02	e3	α=0.07	c3	quintic/PC	g3	20
b4	0.0204	0.02	0.9821	0.72	e4	α=0.03	d	quadratic/V	g4	10
b5	0.0317	0.02	0.6313	1.12	e5	α=0.08				
b6	0.0175	0.02	1.1405	0.62	e6	α=0.02	**Water**	**Horizontal**	**Vertical**	**Bottom**
b7	0.0345	0.02	0.5796	1.22	e7	α=0.09	**Depth**	**Distance**	**Distance**	**Slope**
b8	0.0147	0.02	1.3598	0.52	e8	α=0.06	d=0.34m	l_x=11m	l_z=0.6m	0.05
b9	0.0373	0.02	0.5357	1.32	e9	α=0.01				
b10	0.0402	0.02	0.4980	1.42	e10	α=0.10	**Wave**	**Wave**	**Breaker**	
b11	0.0430	0.02	0.4652	1.52	i	laminar	**Height**	**Period**	**Type**	
b12	0.0571	0.02	0.3501	2.02			H=0.105m	T=2.42sec	Weak Plunger	

Figure 1 illustrates the selection of the latter. The general pattern of conclusion in the analysis of Makris et al. (2009) is still valid, yet somehow enfeebled especially for case b7. In general the SPH-SPS model provides acceptable prediction of the wave height, only for the pre-breaking and inner surf-zone region, while at intermediate gauges, in the vicinity of the experimentally traced breaking point, an excessively over-diffusive performance is still noted. Further decrease of *Δx/h* (cases b11, b12) does not address the issue, specifically in the wave breaking region, where simulations fail totally. The mean surface elevation is, in contrast to the wave height, very well predicted except for the marginal high and low values of *h* in our analysis (Fig.1 – lower graphs). This inconsistency probably hinges on the fact that an inherent unaccounted for discrepancy, due to boundary conditions instabilities in the vicinity of the coastline portrayed previously in Rogers & Dalrymple (2004), expands along the cross-section of the coast bottom with decrease of *Δx/h*, up to the wave generator, as depicted in Figure 2. This censure gives credit to the aforementioned assertion about the optimized spacing to smoothing length ratio *Δx/h*, demonstrating test case b7 calibration as the best fit of our analysis to the SF experimental data.

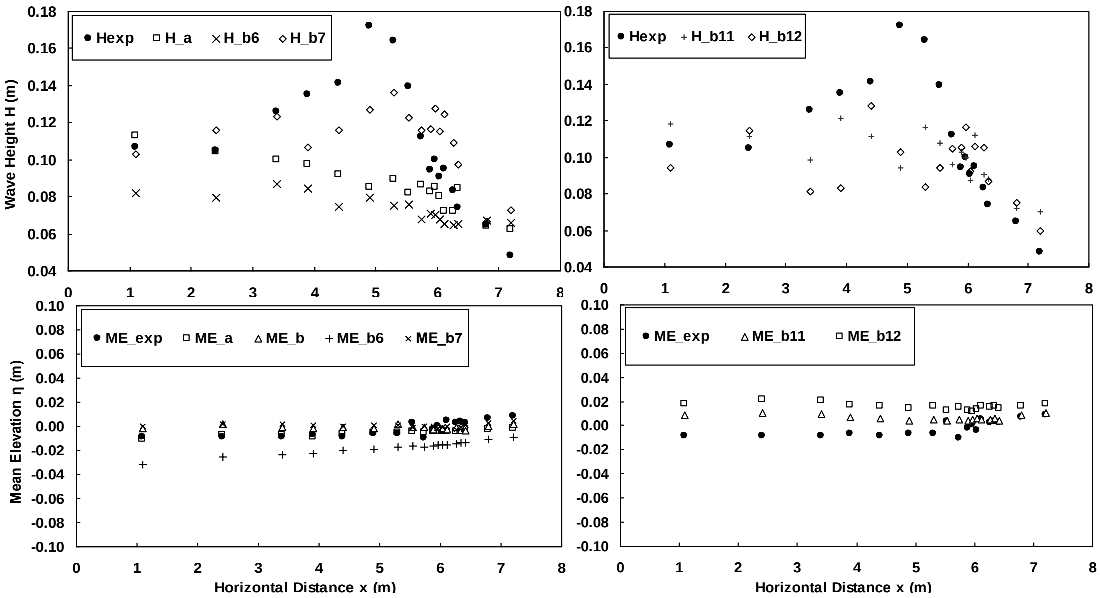

Figure 1. Comparisons of wave height and mean elevation distributions between SF experiments (subscript: exp) and characteristic simulations (test cases: a, b6, b7, b11, b12)

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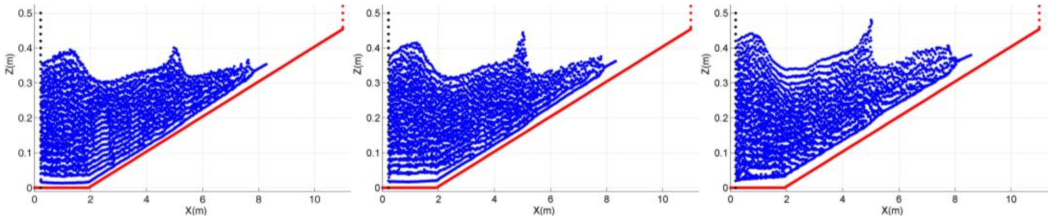

Figure 2. Comparisons of SPHysics deformed axis cross-section output (tests from left to right: a, b7, b12)

An additional remark is that for higher h and consequently lower $\Delta x/h$ values, the visual wave breaking pattern gradually transforms from nearly spilling in test a (Makris et al. 2009) to almost plunging, as actually reported by SF. This is visually verified by the bore propagation sections given in Figure 3 (right panel), where a weak plunger jet formation is traceable, followed by the occurrence of a rebound splash event. Furthermore the increase of smoothing length h causes the magnitude of the maximum velocity at the propagating crest of the breaking wave to become roughly 1.5 times the theoretical value of celerity in shallow water, $c_t = (g\ d)^{1/2}$. This value is reckoned as a better approximation compared to ~ $0.8 \cdot c_t$, computed in test a. Finally, referring to spatial discretization calibration, we can observe (Fig. 3 – left panel) that the finer analysis (Δx, Δz = 0.01 m) invokes higher quality representation of the plunging tongue detachment which in turn is followed by the impinging of the acute abrupted water mass upon the forward trough and consequent rebound splash-up with a secondary bump formation shoreward of the breaking surface roller. This depiction has been performed and analyzed with even more detail by various classic SPH researchers and relates to high values of c_f = 1.52 in our simulations. Further calibration of the smoothing length ratio $\Delta x/h$, combined with the fine analysis is expected to yield even better results.

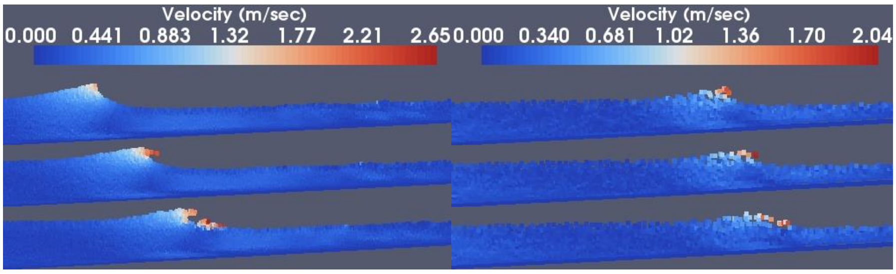

Figure 3. Instantaneous velocity magnitude plot and impinging jet splash-up event of plunging type breaking for test case b7, Δx, Δz = 0.01 m (left panel) and Δx, Δz = 0.02 m (right panel).

As far as viscosity treatment is concerned, it has been argued in the past that the empirical coefficient α constitutes a numerical stability retainer, but becomes practically a factor of excessive dissipation in transient free surface flows (De Padova et al. 2009). Unlike the results of various other researchers, we currently find it difficult to adjust the model's calibration factor α, to cope with the data of SF, no matter what value is assumed inside the reasonable default limits of application (test series e). Of course no combined smoothing factor and artificial viscosity has been attempted in our study yet, a probable eager cooperation to somehow bend the discrepancies. Preliminary results of vorticity gradient comparisons near the wave breaking region among the different implementations shows absence of turbulent production in the incipient breaking region for the SPS model simulations and throughout the whole propagation domain for the artificial viscosity model ones. Visual verification of spilling (not plunging) breaking strengthens the relevant argument also reencountered in the default (test a) simulation (Makris et al. 2009).

In relation to the influence of different numerical schemes and kernels to the quality of computations, we only succinctly mention here that the higher the order of the numerical scheme (Table 1 – test series c) the more elaborate the computations turn to be. Nevertheless they do not reach the levels of accuracy obtained by calibration of the smoothing length. Thus combination of the two approaches should reveal the optimum results. Furthermore, the use of the Verlet type algorithm (test d) may not exhibit significant progress in terms of accuracy. On the other hand it accelerates the computations by a factor of almost two. Finally the reduction of compressibility factor c_B affords a little higher quality computational attributes, but exhibits a certified lower limit value of 10 for reasonable artificially compressible simulations according to the Prandtl-Glauert rule (Monaghan 2005; Makris et al. 2009). Further detailed publication on the above matters is being currently prepared.

5 CONCLUSIONS

The recently brought out 'open source' academic code SPHysics (Gómez-Gesteira et al., 2007) setting the computational framework of implementation for a promising ingenuous CFD mesh-free numerical method called SPH is being calibrated against experimental data of wave propagation and breaking on smooth mild sloping beaches placed inside a laboratory scale wave flume by SF. Plausible agreement is achieved in terms of wave heights and bore front velocities, for one optimum dimensionless smoothing ratio. The combined use of the best numerical schemes, computational kernels and artificial compressibility manipulation seems promising and is planned for future research. In general, prediction of wave heights is still acceptable only in the pre-breaking and inner surf zones. The wave breaking process is somehow overestimated in the incipient and mid-breaking region, with a consequent underestimation of wave height there. Nonetheless surely better results are provided in the framework of our calibration proposals. The SPS Smagorinsky-type eddy viscosity model used for the closure of turbulence, in the type of LES-SGS models seems to treat the turbulent energy cascade from resolved to unresolved scales rather poorly, especially in areas with great velocity gradients. Moreover backscatter phenomena are being neglected and the artificial viscosity concept proves rather insufficient to address the issue, despite the calibration endeavours. Conclusively the latest and incoming results of our analysis enhance the argument that a dynamic Smagorinsky-type model, based on Germano's identity (Germano et al. 1991, Lilly 1992), which takes into account the evolution of the ambient velocity field, proves to be compulsory for SPH simulations and is proposed for prospective research. Further implementation of 3-D simulations, are expected to enlighten the turbulent features in the surf zone.

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